Should You Kill or Embrace Your Competitor: Cloud Service and Competition Strategy

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Cloud services have grown rapidly in recent years as a new form of service, but there exists scant research on the competition and service offering strategies of cloud service providers. We study these strategic choices by considering a setting where an incumbent cloud service provider may offer two service classes (premium and standard) and an entrant provider may offer the standard service. We analyze two competitive strategies the incumbent provider: (a) deterring competitor entry, and (b) not deterring competitor entry. We also evaluate two service offering strategies of the incumbent provider: (a) offering both the premium and the standard service, and (b) offering the premium service only. Based on results of our model, we present several useful managerial insights. First of all, we find that, under monopoly setting, it may sometimes be beneficial for the service provider to encourage higher usage despite the extra cost. Further, in the duopoly setting, higher customer usage impacts the Deter/Allow strategy choice and is neither always beneficial nor always detrimental to the incumbent provider, and such impact depends on profit margin and customer usage distribution. We also find that a larger market encourages the incumbent provider to add the standard service and switch between Allow and Deter strategies, since a larger market size may reduce the profit of an incumbent provider sticking to the Deter strategy. On the capacity side, we discover that the incumbent provider should strive to deter entry threats when it is at a cost advantage in a small market.

Key words: cloud services, entry deterrence, service differentiation, game theory, capacity.

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“Competition-to-the-almost-death seems the norm in SaaS.”

—Jason Lenkin (Lenkin 2012)

1 Introduction

With ever-increasing R&D investment and customer adoption in recent years, cloud services are “the future of business” (Loten 2015a). Cloud services represent a paradigm shift in the ways services are made available to businesses and individuals, where customer needs are satisfied remotely over the Internet (Fox et al. 2009). International Data Corporation (2015) predicts end-user spending on cloud software reaching $48.8 billion in 2014 and growing to surpass $112.8 billion by 2019 at a compound annual growth rate of 18.3%. According to a recent survey by Verizon, roughly half of the companies believe that at least 75% of their workloads will be in the cloud by 2018 (Loten 2015b). Because of the promising future, the competition among cloud service providers is becoming “particularly fierce” and “even more difficult than it was a year ago” (PricewaterhouseCoopers 2014). For instance, Hewlett-Packard Co. winded down its Helion Public Cloud in October 2015, which competes with Amazon’s AWS service (McMillan 2015). The intensified competition calls for answering one question: What is the best strategy for cloud service providers under competition? The focus of this paper is to address this question.

1.1 Problem Motivation

In this study, we focus on a particular form of cloud service, Software-as-a-Service (SaaS). Instead of owning perpetual licenses of the software, SaaS customers pay a fixed monthly or annual subscription fee to gain access to the software application, often without one-time upfront expenditure (Ma and Kauffman 2014). Unlike other forms of cloud service, i.e. IaaS (Infrastructure-as-a-Service) and PaaS (Platform-as-a-Service), SaaS customers generally enjoy free service usage once the subscription fee is paid. SaaS has quickly become a major delivery mode of service, projected to grow from $49 billion in 2015 to $67 billion in 2018 (Columbus 2015). Cisco (2015) predicts that by 2019, 59% of the total cloud workloads will be SaaS-based workloads, up from 45% in 2014. Various SaaS
applications, such as cloud storage and music streaming, are becoming increasingly popular among consumers and businesses (International Federation of the Phonographic Industry 2014).

Similar to most cloud services, competitive pressure is paramount on the shoulders of every incumbent SaaS provider as entry threats emerge. As stated by Jillian Mirandi, an analyst at Technology Business Research: “In the fast-growing SaaS market, vendors want to expand their customer footprint before their competitors do” (Kwang 2012). This leads to fierce competition as commented by Jason Lemkin, an SaaS venture capitalist: “Competition-to-the-almost-death seems the norm in SaaS” (Lemkin 2012). Despite growing academic interest in SaaS competition (e.g., Ma and Kauffman 2014), there remain many unresolved managerial challenges pertaining to entry threats. For example, the dominant provider in customer relationship management (CRM) cloud service, Salesforce.com, faces recent entry by Microsoft (Kim 2015). As another example, the launch of Apple Music poses serious threats to music streaming providers in many countries (Dredge 2015), calling for appropriate strategic and operational decisions of the incumbent providers. Driven by this need, we analyze the competition and service strategies of SaaS providers with a game-theoretic approach. We also seek to understand the role of capacity in this context.

The leader in creative software and maker of Photoshop, Illustrator, etc., Adobe Systems Inc., made the decision to stop selling boxed-software for many of its product lines in 2013, transforming itself into a cloud service provider and becoming a natural monopolist in this market (Jones 2013). However, Adobe’s competitors may soon follow suit and enter this cloud service market, meaning that Adobe needs to choose whether to deter potential entrants or not. The choice faced by Adobe is not uncommon, and cloud service providers vary in their responses. On one hand, the dominant provider should “identify and preempt market opportunities before new market entrants can gain a foothold,” as suggested by Bill Schmarzo, an executive at EMC Inc. (Schmarzo 2013, p. 103). On the other hand, Spotify strove to strengthen itself as the leader in the music streaming market amid market entries of Apple and Google (Dredge 2014), meaning that not deterring competitor entry may be preferred in certain cases.
This Deter/Allow strategic choice is further complicated by the subscription pricing frequently used in cloud services (e.g., Microsoft Office 365, Netflix, and Apple Music) and the providers’ need to price competitively to attract customers (Dignan 2012). Many consumer-oriented cloud services adopt subscription pricing due to consumer preference and low transaction cost of flat-fee pricing (Sundararajan 2004, Lambrecht and Skiera 2006, Lambrecht et al. 2007). “There’s no question the pendulum will continue to swing to subscriptions,” as commented by Nick Davies, general manager of Corel’s digital media and productivity software group (Shankland 2012). Hence, for subscription-based cloud services, it is our research goal (and of great importance for the providers) to understand how the Deter/Allow strategic choice under subscription pricing can differ from traditional business model. This difference follows from the fact that higher service usage adds to the capacity cost but not necessarily adds revenue to the provider under subscription pricing, unlike traditional pay-per-use pricing where customers are charged by usage.

Our next research goal involves service offering differentiation. Customers of cloud services demand high quality of service (QoS). In practice, service level agreement (SLA) are routinely used to guarantee service levels to customers (The European Commission 2014). One of the important dimensions of service level is service delay (Groom and Groom 2004, p. 21) since users of online services are highly sensitive to delays (Hamilton 2009, Kohavi et al. 2009, Lohr 2012). For example, even undercutting competitor’s response time by as little as 250 milliseconds is considered a unique advantage by Microsoft (The New York Times 2012), and slowness can be strategically costly (Moallemi and Saglam 2013, Singla et al. 2014). Responding to this need, service providers may differentiate their service offering in QoS to cater to various customers (Zhang et al. 2007, 2009). While providing differentiated services may create a competitive edge (Allon and Federgruen 2007, Fan et al. 2009), service differentiation may lead to cannibalization and hurt the provider’s profit. This dilemma over service differentiation is worth studying.

1.2 Research Questions and Our Contributions

In this paper, we start with the monopoly scenarios to gain insights that pave way to analyzing duopoly scenarios. We first ask the following question: Is higher customer usage beneficial or hurtful
for the cloud service provider? In subscription services, one would expect higher customer usage to always be hurtful. However, we find that it is not always so in our setting. In fact, under certain scenarios, it may be beneficial for the service provider to encourage higher usage.

Further, as discussed earlier, another important question we analyze is: Should a cloud service provider offer multiple services? By answering this research question, our study adds to the service operations literature (e.g., Allon and Federgruen 2009, Zhao et al. 2012) by identifying various factors contributing to the service differentiation decision. We investigate the service differentiation decision in a scenario where the entrant provider offers a standard service while the incumbent provider features a premium service and has the option to offer a standard service as well. Managers may be interested in understanding the trade-off over service differentiation under subscription pricing where consumer usage need to be taken into account, unlike traditional pay-per-use service pricing. Since insufficient capacity may cause significant delays (Mullaney 2013), managers may also wonder how capacity cost difference between different service offerings impact both the Deter/Allow strategic choice and the service differentiation choice of the incumbent provider, and our paper helps to enhance understanding in this direction.

Towards the research question on service offering, in monopoly settings, we show that both the larger market size and the lower capacity cost difference between different service offerings encourage more classes of service, stressing the need to launch new services when business conditions change. We also identify conditions when it is beneficial to offer only one class of service. These results are similar to those of Zhang (2009) in the sense that having multiple classes of service may not always be the best strategy. We also discover that customer valuation of the service does not impact service offering decisions. In duopoly settings, we find that offering the standard service in addition to the premium service helps in deterring competitor entry; however, the benefits from entry deterrence may not always justify service cannibalization that hurts the profit.

As discussed earlier, another important question that we analyze is: Should the incumbent firm Deter or Allow the entrant in cloud service market? For this question, the answer is not straightforward since these two strategies form a difficult trade-off: The former strategy entails lower pricing
and higher capacity cost while the latter strategy often leads to a smaller market share and hence lower revenue. Managers may be interested in understanding factors impacting this strategic choice such as market and firm characteristics. We find that the Deter/Allow strategy choice has never been fully revealing, and a thorough analysis is needed to understand different trade-offs. This is one of the key focus of our paper.

A related question is: *Will a larger market size always be beneficial if the incumbent provider sticks to the Deter strategy?* Our answer is no, which joins Zhang et al. (2009) in showing that service differentiation of the incumbent service provider happens only when market size is big enough to justify the capacity cost. Another question we explore is: *Will higher usage always be beneficial to the incumbent provider?* We show that this is not necessarily the case under monopoly, and we also find that the usage distribution of consumers impacts the Deter/Allow strategy choice.

We also examine whether firm and consumer characteristics, such as unit capacity cost, customer valuation of service, and usage, impact the optimal/equilibrium strategy. We demonstrate that cloud service providers should respond to changes in these characteristics with an appropriate strategy, and our analysis would assist cloud service providers to forecast future sales and capacity levels before rolling out services in new markets and to assess the opportunities in add-on services based on consumer characteristics of the original service.

This paper adds to the service competition literature (e.g., Allon and Federgruen 2007, Seamans 2012, Wang et al. 2016) by showing that under subscription pricing, it is important to understand customer heterogeneity thoroughly and adopt strategies that single out profitable customers while making the service less attractive for unprofitable customers. In particular, we demonstrate that customer usage may impact the Deter/Allow strategy choice with moderate usage favoring the Deter strategy, meaning that managers should monitor the service usage and prepare for a strategy shift if usage changes. Our findings also differ from studies in online channel entry such as Liu et al. (2006) where a brick-and-mortar retailer may deter online entry by refraining from online channel in the presence of positive entry cost. We show that while it might be possible and even optimal to
deter competitor entry using the premium service only, using both standard and premium services
to deter competitor entry could be a better choice in many circumstances.

Besides these research questions, one research goal of our paper is to understand the role of sub-
scription pricing that charges no fee on incremental usage. We find that higher usage by customers
is neither necessarily beneficial nor necessarily detrimental to the incumbent provider. Related to
this finding, Zhang et al. (2007) show that higher usage generally brings more profit to the service
provider unless significant congestion occurs. Since our paper differs from Zhang et al. (2007) by
considering subscription pricing, additional revenue brought by higher usage does not necessarily
compensate the increase in capacity cost unless the per-use value of the service is sufficiently high,
which encourages service providers to enhance the value of their service offering.

We also investigate the impact of capacity cost in cloud service, which can be significant. Sab-
harinath Bala, a research manager at IDC, comments: “Unlike traditional on-premise solutions,
SaaS offerings have much higher operating costs as percentage of revenue, which has resulted in
diminishing the profit margin for the SaaS business model” (Kwang 2012). Our paper provides
several findings relating to capacity cost differences between service classes and firms, which may
impact competitive and service offering strategies. For example, the unit capacity cost may influ-
ence whether a strategy is feasible or preferred. Moreover, depending on relative competitiveness of
different service offerings, the incumbent provider may choose to deter or allow competitor entry.
For example, an across-the-board increase in capacity cost favors the Deter strategy, and a higher
capacity cost differential between the entrant provider and the incumbent provider favors the
Allow strategy. While both changes in capacity cost enhance the competitiveness of the incumbent
provider, they have different implications on the competition strategy.

1.3 Related Literature

Our work is broadly related to three streams of literature: Information goods pricing, entry-
deterrence, and service competition. In this subsection, we briefly discuss the related work in these
streams. Here, we also compare and contrast our work with these studies in order to highlight our
contributions.
1.3.1 Information Goods Pricing: Pricing of information goods stemmed from internal pricing of computing resources with queue delays (Mendelson 1985) and later extended to Internet service pricing. Many of these studies ignore capacity (Bashyam 2000) or consider capacity as a constraint (Essegaier et al. 2002), and hence they cannot analyze endogenous capacity decisions, a key aspect of our model. Some information goods pricing studies incorporate customer heterogeneities: Bala and Carr (2005) consider heterogeneities in customers’ per-use valuation and usage, but do not include capacity considerations as we do in this paper; Niculescu et al. (2012) also model customer differentiation in value of service but bring network benefits into consideration, thus differing in focus with our paper. Different from most studies in information goods pricing, we consider subscription pricing in this paper and examine the strategic intricacies it brings.

1.3.2 Entry-deterrence: Economists have long been interested in entry-deterrence (e.g., Wilson 1992), but they mostly ignore actual production and/or service processes. Some retail studies investigate how an appropriate channel structure can gain market advantage, including deterring online competitor entry (Chiang et al. 2003, Liu et al. 2006, Zhang 2009, Yoo and Lee 2011). While also exploring entry-deterrence strategies, our paper differs in focus with this literature by considering services delivered online rather than physical goods with inventory issues.

1.3.3 Service Competition: Different from inventory-focused competition (Lederer and Li 1997), our paper builds on extant studies in service competition where price and capacity are often the decision variables (e.g., Hall and Porteus 2000, So 2000, Boyaci and Ray 2006, Allon and Federgruen 2009). Service competition has also attracted interest in the context of IT services. Zhang et al. (2007) study the monopoly pricing of communication services with explicit QoS guarantees of expected delay. Cheng et al. (2003) and Zhang et al. (2009) analyze capacity and pricing decisions of duopolistic Internet service providers, while Ma and Kauffman (2014) focus on the switching cost of IT service clients. Similar to these studies in service competition, we consider price competition between heterogeneous providers where capacity decisions and capacity costs are relevant to strategy choices. However, unlike most studies in this research stream, we
adopt subscription pricing and incorporate customer heterogeneities in delay sensitivity and usage explicitly, which allows us to explore the connection between service revenue and capacity cost in subscription-based cloud services. Moreover, our paper investigates the effect of customer usage on market segmentation and considers a sequential setting to analyze entry-deterrence strategies. While considering differentiated services between providers as some extant studies do, we find cases where the incumbent provider can gain both maximal profit and entry-deterrence by offering services matching with the entrant’s services.

Some operations management researchers explore service competition under entry threats. As a recent example, Wang et al. (2016) explore how the incumbent firm prices its products and allocates its limited capacity across markets under entry threats. Other examples include Cournot competition (Anand and Girotra 2007) and re-manufacturing (Ferguson and Toktay 2006). While also examining entry-deterrence decisions in pricing and capacity, our paper emphasizes service quality and subscription pricing, thus differing in focus.

The rest of this paper is organized as follows. Section 2 introduces the model assumptions. Section 3 analyzes a monopoly cloud service provider with single- and dual-classes of service. Section 4 considers a potential entrant with a single class of service in addition to the incumbent provider and analyzes the Deter/Allow strategy choice of the incumbent provider. Section 5 concludes the paper and provides directions for future research.

2 Model and Assumptions

In this section, we introduce the game-theoretic model and our assumptions, with key notations summarized in Table 1. Proofs of all lemmas and propositions are provided in the Online Appendix.

2.1 Firms

To capture the market dynamics in cloud service, we consider two firms: an incumbent provider and an entrant provider. In duopoly scenarios, the two providers are represented by the subscripts $i$(incumbent) and $e$(entrant). Similar to Zhang et al. (2009) and analogous to the product offerings
Table 1  Key notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
<th>Comments</th>
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<tbody>
<tr>
<td>$j$</td>
<td>class of service, $j = 1, 2, 3$</td>
<td></td>
</tr>
<tr>
<td>$d_j$</td>
<td>Average delay of class-$j$ service</td>
<td></td>
</tr>
<tr>
<td>$p_j$</td>
<td>Price of subscribing to class-$j$ service$^{2,3}$</td>
<td>Decision variables</td>
</tr>
<tr>
<td>$e_j$</td>
<td>Unit capacity cost of class-$j$ service</td>
<td></td>
</tr>
<tr>
<td>$N_j$</td>
<td>Number of subscribers of class-$j$ service$^{2,3}$</td>
<td></td>
</tr>
<tr>
<td>$U_j$</td>
<td>Amount of usage of class-$j$ service$^{2,3}$</td>
<td></td>
</tr>
<tr>
<td>$u_j$</td>
<td>Capacity level of class-$j$ service$^{2,3}$</td>
<td>Decision variables</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Customer’s delay sensitivity</td>
<td>Subscripts denote different values</td>
</tr>
<tr>
<td>$k$</td>
<td>Incremental utility for each unit of usage</td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>Customer’s usage</td>
<td></td>
</tr>
<tr>
<td>$m_0$</td>
<td>Minimum usage</td>
<td>Parameter in the usage distribution</td>
</tr>
<tr>
<td>$b$</td>
<td>Customer usage shape</td>
<td>Parameter in the usage distribution</td>
</tr>
<tr>
<td>$v_0$</td>
<td>Utility for having access to any service</td>
<td></td>
</tr>
<tr>
<td>$v_j$</td>
<td>Utility of a customer of class-$j$ service</td>
<td>Customer utility: $v = \max{v_1, v_2, v_3, 0}$</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Profit of a provider$^{1,2}$</td>
<td></td>
</tr>
<tr>
<td>$q$</td>
<td>Unit capacity cost differential</td>
<td>The standard service only; $q = e_3/e_2$</td>
</tr>
<tr>
<td>$a$</td>
<td>Market size</td>
<td></td>
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</table>

$^1$ Subscripts $i$ and $e$ denote the incumbent provider and the entrant provider, respectively.

$^2$ Subscripts $a$ and $b$ denote Premium-only and Dual-class, respectively.

$^3$ Superscripts $d$ and $nd$ denote Deter and Allow, respectively.

of Bazaarvoice Inc. after its acquisition of PowerReviews Inc. (Bazaarvoice 2012), we consider that the incumbent provider offers a premium (class-1) service and has the option to offer a standard (class-2) service to capture the effect of service differentiation. Clearly, the premium service has a
higher price and a lower service delay than the standard service. The entrant provider offers only a standard (class-3) service that has the same service delay with the standard (class-2) service offered by the incumbent provider. This is analogous to the attempt by Corel Inc. to compete with Adobe Inc. in the creative software subscription market with a lower-quality service.

We use subscript $j$ to denote the service class. The average delay of a service class is denoted as $d_j$ with $d_1 < d_2$ and $d_3 = d_2$. The subscription fees (i.e., price) of each service class are denoted as $p_j$ with $p_2 < p_1$. The capacity level of class-$j$ service is denoted as $u_j$ with the unit capacity cost $e_j$. Moreover, similar to Goyal and Netessine (2007), we allow the firms to differ in capacity cost, where the unit capacity cost of the entrant provider is $q$ times that of the incumbent firm (i.e., $e_3 = qe_2$). We use $N_j$ to denote the number of subscribers in each service class and $U_j$ to denote the amount of usage in each service class. Similar to that in practice, we consider that the prices and service delays are common knowledge among providers and consumers, whereas the unit capacity costs are known among providers.

2.2 Consumers

Similar to that in past studies (e.g., Bala and Carr 2005, Zhang et al. 2009, Niculescu et al. 2012), we consider that a user’s disutility from delays is proportional to usage, and that the value of the cloud service to a user is linear in usage.

Following Bala and Carr (2005), there is a continuum of potential users who vary in usage (denoted as $m$) and delay sensitivity (denoted as $\theta$), where $\theta$ is uniformly distributed over the interval $[0, 1]$. Cloud resource usage are typically distributed with tails heavier than log-normal, exponential, and normal distribution (Loboz 2012), where the proportion of users with certain usage diminishes quickly as usage increases. Similar patterns can be found in YouTube usage (Gill et al. 2008), smart phone usage (Falaki et al. 2010), and broadband Internet usage (Federal Communications Commission 2014, pg. 51). Hence, we employ a heavy-tail distribution, the Pareto distribution, to model the distribution of usage across potential subscribers: the probability density
function of usage is \( f(m) = \frac{bm^b}{m^b + 1} \), where the minimum usage is denoted as \( m_0 \) and the distribution of usage is characterized by the parameter \( b > 1 \).

Each user receives utility \( v = v_0 + km \) from using the cloud service, where \( v_0 \) is the utility for having access to the service and \( k \) is the marginal utility gained per unit usage. The net utility of a user of class-\( j \) service is therefore \( v_j = v_0 + km - p_j - d_j \theta m \).

2.3 Firm and Consumer Decisions

As discussed in Section 1.2, we consider a game-theoretic setting where service delay guarantees are exogenous, such as voice applications (International Telecommunications Union 2003, Chen et al. 2004). Similar to the sequential game in Wang et al. (2016), the game proceeds in three stages to capture the strategic behavior among the incumbent provider, the entrant provider, and individual consumers. We illustrate the game setting in Figure 1 and outline the strategies of the incumbent provider in Table 2 with further discussions in this subsection. We solve the game using a backwards induction procedure (Mas-Colell et al. 1995, p.277) to identify the subgame perfect Nash equilibrium.

**Stage 1:** The incumbent provider moves and decides capacity levels \((u_1, u_2)\), after which the prices \((p_1, p_2)\) are announced. As previously mentioned, both the Deter strategy (e.g., Adobe) and the Allow strategy (e.g., Salesforce.com) can be chosen, and service differentiation is also an option under the Deter strategy (e.g., Bazaarvoice).

**Stage 2:** The entrant provider decides whether or not to enter the market in response to the incumbent provider’s decisions. If it decides to enter, capacity level \((u_3)\) will be determined and the price \((p_3)\) will be announced. This stage only applies to scenarios with entry threat (duopoly) and captures the strategic interactions associated with the market entry decision. An example of the entrant provider is Oracle, which entered the CRM SaaS market long after Salesforce.com had dominated this market.

**Stage 3:** The consumers collect information about service offerings in the market and decide on their subscription. A consumer makes the service choice (class-1, class-2, class-3, or no service)
Table 2  Scenarios in this paper

<table>
<thead>
<tr>
<th>The Incumbent (The Entrant)</th>
<th>Deter (Refrain)</th>
<th>Allow (Enter)</th>
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<tbody>
<tr>
<td>Premium-only</td>
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<td>Salesforce.com</td>
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The Incumbent

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Table 2: Scenarios in this paper

The Incumbent (The Entrant)

Deter (Refrain)  

Allow (Enter)  

The Incumbent

Premium-only  
Adobe  
Salesforce.com  

Dual-class  
Bazaarvoice  
Dominated by Premium-only Allow  

to maximize her utility \( v = \max\{v_1, v_2, v_3, 0\} \) after the firms complete their moves, which follows from the usual incentive-compatibility constraint. A consumer does not subscribe to any service if all service classes provide negative utility for her, which is the individual rationality constraint.

Figure 1  The game setting

3 Monopoly Scenarios

In this section, we begin with the monopoly scenario. The purpose of this analysis is two-fold: First, many service industries (such as telecommunications) are monopoly or near-monopoly (The
Economist 2012, Crawford 2013), which makes our results widely applicable; a good example in cloud services is Adobe Systems Inc. as mentioned earlier. Second, the results of this analysis may provide us some directions and insights for analyzing the duopoly scenario. In this section, we also analyze the implications of subscription pricing and service differentiation, focusing on the impact of various factors such as customer usage and valuation of the service.

3.1 A Single Service Class

In this subsection, we consider a scenario where a monopolist cloud service provider offers only the premium service. It follows from $v_1 = v_0 + km - d_1 \theta m - p_1 \geq 0$ that $\theta_1 = \frac{km + (v_0 - p_1)}{d_1 m} \geq 0$ is the upper-limit of delay sensitivity of potential customers with usage $m$ that choose the class-1 service.

When $\theta > \theta_1$, the customer will not use the service. Hence, the number of subscribers is

$$N_1 = \int_{m_0}^{+\infty} [f(m) \cdot \theta_1] dm.$$

Similarly, the total usage is

$$U_1 = \int_{m_0}^{+\infty} [m f(m) \cdot \theta_1] dm.$$

The incumbent firm’s profit maximization problem can now be written as:

$$\max_{(p_1, u_1)} \Pi = N_1 \cdot p_1 - e_1 \cdot u_1;$$

$$s.t. u_1 \geq U_1 + 1/d_1.$$

We summarize the optimal decisions of the provider in Lemma 1.

**Lemma 1.** Under the monopoly scenario with a single service class:

(a) The optimal price in this scenario is $p_{1a} = \frac{(1+b)m_0(e_1+k)+bv_0}{2b}$.

(b) The optimal capacity level is $u_{1a} = \frac{l^2[(a v_0 + 2 - a m_0 (e_1 - k)) - b(a v_0 + 2) + a m_0 (e_1 + k)]}{2(b-1)bd_1}$.

(c) The optimal profit is

$$\Pi_a = -\frac{a \left( (b+1)^2 \epsilon_1 m_0^2 - (b+1)m_0 k + bv_0 \right)}{4bd_1(b+1)m_0} + \frac{2e_1 \left( l^2 (a m_0(e_1 - k) - a v_0 - 2) + b(a v_0 + 2) - a m_0(e_1 + k) \right)}{4bd_1(b-1)}.$$

It follows from Lemma 1 that higher usage increases both price and capacity level. However, it is unclear how the profit reacts to increase in customer usage, which we will study in Section 3.3.
3.2 Two Service Classes

In this section, we examine the scenario where two service classes are provided by a monopolistic service provider, such as Bazaarvoice Inc. after its acquisition of PowerReviews (Bazaarvoice 2012). The added standard service is an attractive option for low-usage, low-delay-sensitivity customers, and it may improve the market coverage of the monopolistic provider. Similar to Ma and Kauffman (2014), we use two classes of service to study the impact of service differentiation.

It follows from $v_1 = v_0 + km - d_1 \theta m - p_1 \geq v_0 + km - d_2 \theta m - p_2 = v_2$ that $\theta_3 = \frac{p_1 - p_2}{(d_2 - d_1)m} \geq 0$ is the upper-limit of delay sensitivity of potential customers with usage $m$ that choose class-1 service over class-2 service. When $\theta > \theta_1$, a customer will not use the service. Hence, the number of subscribers of the premium service is

$$N_{1b} = \int_{m_0}^{+\infty} [f(m) \cdot (\theta_1 - \theta_3)] dm,$$

and the total usage of the premium service is

$$U_{1b} = \int_{m_0}^{+\infty} [mf(m) \cdot (\theta_1 - \theta_3)] dm.$$

Similarly, the number of subscribers of the standard service is

$$N_2 = \int_{m_0}^{+\infty} [f(m) \cdot \theta_3] dm,$$  \hspace{1cm} (1)

and the total usage of the standard service is

$$U_2 = \int_{m_0}^{+\infty} [mf(m) \cdot \theta_3] dm.$$  \hspace{1cm} (2)

The incumbent firm’s profit maximization problem can now be written as:

$$\max_{(p_1,p_2,u_1,u_2)} \Pi = N_{1b} \cdot p_1 - e_1 \cdot u_1 + N_2 \cdot p_2 - e_2 \cdot u_2;$$

$$s.t. \quad u_1 \geq U_{1b} + 1/d_1;$$

$$u_2 \geq U_2 + 1/d_2.$$

We outline the optimal solution in Lemma 2.
Lemma 2. Under the monopoly scenario with two service classes:

(a) The optimal prices are:
   
   (i) \( p_{1b} = \frac{(1+b)m_0(e_1+k)+b\nu_0}{2b} \) for the premium service.
   
   (ii) \( p_{2b} = \frac{(1+b)m_0(e_2+k)+b\nu_0}{2b} \) for the standard service.

(b) The optimal capacity levels are:
   
   (i) \( u_{1b} = \frac{b^2(d_1(ae_2m_0-akm_0-\nu_0-2)+d_2(a(-e_1m_0+km_0+\nu_0)+2))}{2(b-1)bd_1(d_2-d_1)} + \frac{b(d_2-d_1)(-\nu_0-2)+am_0(d_2(e_1+k)-d_1(e_2+k))}{2(b-1)bd_1(d_2-d_1)} \)
   for the premium service.

   (ii) \( u_{2b} = \frac{1}{d_2^2} + \frac{am_0(b+1)(e_1-e_2)}{2b(d_2-d_1)} \) for the standard service.

It can be shown that cannibalization between the two service classes exists under this scenario.

The expression of the profit is tedious, and therefore reported in the Online Appendix for brevity.

3.3 Results and Managerial Insights

In this subsection, we explore the impact of service usage and market size on profit and service
differentiation under monopoly.

3.3.1 Minimum Usage: In Proposition 1, we first explore the effect of minimum usage on
profit with only the premium service.

Proposition 1. The effect of minimum usage on profit under monopoly with only the premium
service can be characterized as follows:

(a) When \( k > \frac{b+1}{b-1}e_1 \), there exists a threshold of minimum usage \( \tilde{m}_0 \) such that

   (i) When \( m_0 > \tilde{m}_0 \) and \( \nu_0 > 0 \), the monopoly profit increases in \( m_0 \) and vice versa, where

   \[ \tilde{m}_0 = \frac{\sqrt{(1+b)[(b+1)e_1-(b-1)k][-(b-1)e_1-(1+b)k]}}{\sqrt{(1+b)[(b+1)e_1-(b-1)k][-(b-1)e_1-(1+b)k]}}. \]

   (ii) When \( \nu_0 = 0 \), the monopoly profit always increases in \( m_0 \).

item When \( k < \frac{b+1}{b-1}e_1 \), the monopoly profit always decreases in \( m_0 \).

The outcome of Proposition 1 is that higher minimum usage (and hence higher overall usage) does not necessarily reduce the profit and may actually improve the profit. The intuition is that individual customer usage matters under subscription pricing, unlike under fee-for-service pricing.
where every unit of usage gets paid. On the one hand, as the usage increases, the firm is able to charge higher price (see Lemma 1) and attract more subscribers; thus increasing the revenue of the firm. On the other hand, more usage increases the capacity cost for the firm. When the firm provides sufficient value to customers (i.e., \( k > \frac{b+1}{b-1}e_1 \)) with high usage (\( m_0 > \hat{m}_0 \)), the increase in revenue outweighs the increase in capacity cost as the minimum usage increases. However, when the usage is low (\( m_0 < \hat{m}_0 \)), the increase in capacity cost outweighs the increase in revenue as the minimum usage increases. A special case is zero access utility (\( v_0 = 0 \)) where customers become subscribers only due to usage, and thus higher usage leads to higher profit. However, if value provided to customers is insufficient (\( k < \frac{b+1}{b-1}e_1 \)), the benefit of having higher usage cannot compensate the cost of capacity addition and consequently hurts the profit. Note that the results similar to Proposition 1 can also be obtained for the monopoly scenario with two classes of service (discussed in Section 3.2). However, since the insights are similar, the details are omitted for brevity.

Based on Proposition 1, an important lesson to learn is that the value provided to customers matters in SaaS, perhaps more than what most of the people think. Without sufficient value to customers, higher usage can be a curse for a provider, even a monopolistic one, primarily due to the not-so-direct link between usage and revenue. With sufficient value to customers, higher usage is not always detrimental to the firm’s profit and may be desirable under many situations. It follows that, to benefit from higher usage, firms need to build services of sufficient value to users (\( k > \frac{b+1}{b-1}e_1 \)) and encourage the users to get over the bump (\( m_0 > \hat{m}_0 \)). Otherwise, firms may find themselves in a trap where higher usage hurts the profit.

### 3.3.2 Usage Distribution:

In Proposition 2, we now focus on the impact of customer usage distribution on the optimal profit with two classes of service. For simplicity of exposition, in the following proposition, we consider equal unit capacity cost across service classes (\( e_1 = e_2 \)) and no access utility (\( v_0 = 0 \)) with the following condition on the unit capacity cost: \( k < (7 + 4\sqrt{3})e_2 \). This condition is satisfied when the capacity cost is not negligible compared to the service value, i.e., \( \frac{e_2}{k} > \frac{1}{7+4\sqrt{3}} \approx 0.072 \). This represents a realistic scenario, because, as discussed in Section 1.2, the impact of capacity cost in cloud service can be significant (Kwang 2012).
Proposition 2. When $e_1 = e_2$, $v_0 = 0$, and $k < (7 + 4\sqrt{3})e_2$, there exists a threshold of the usage distribution parameter $b$, which we denote as $\hat{b}$. If $b > \hat{b}$, the optimal profit decreases in $b$, and vice versa, where $\hat{b} = \frac{e_2^2 + 2e_2k + k^2 + 2(e_2 + k)\sqrt{e_2k}}{(e_2 - k)^2}$.

The outcome of the above result is that the distribution of usage among potential users does impact the optimal profit, and that more uneven distribution is not necessarily bad for the monopolist provider. The intuition is that, while an increase in $b$ reduces the proportion of high-usage users, it also leads to a reduction in overall usage. On the one hand, having fewer heavy users lowers the required capacity level and thus reduces total capacity cost; on the other hand, reduction in usage leads to lower customer utility and thus reduction in the number of subscribers and/or a shift from the premium service to the standard service. When $b$ is high (i.e., $b > \hat{b}$), the decline in subscription outweighs the reduction in capacity cost, and thus hurts the profit as $b$ increases. This is because, although heavy-users consume more capacity than average users, they are more likely to become subscribers due to their high overall valuation of the service. When $b$ is low (i.e., $b < \hat{b}$), the reduction in capacity cost outweighs deterioration in subscription, and thus helps the profit as $b$ increases. Our extensive experiments (presented in the Online Appendix) show that the qualitative results and key insights of Proposition 2 do not change for a general case with unequal unit capacity cost and non-zero access utility.

Managers should be prepared when the usage of potential users shift towards more/less skewed, and evaluate how such shift impacts profit. Managers may also consider altering the usage distribution to their benefit if such opportunities present themselves (e.g., encouraging light users to increase their usage or heavy users to reduce their usage), but a thorough analysis is needed to avoid unintended consequences on profit.

3.3.3 Market Size, Service Differentiation, and Consumer Surplus: We now have Proposition 3 regarding the impact of market size and other factors.

Proposition 3.
(a) The monopolist provider improves its profit by introducing the standard service only when the market size is larger than a threshold, i.e., \( a > \hat{a} = \frac{4b(c_2 - c_1)}{m_0d_2(b+1)(c_1 - c_2)^2} \).

(b) The incremental consumer surplus brought by introducing the standard service is
\[
a(1+b)(c_1 - c_2)^2 \frac{m_0}{b(d_2 - d_1)^2} \text{, which is non-negative.}
\]

Two effects demonstrate themselves in Proposition 3(a). First, when the market size is high, advantages of market segmentation is high, which helps to extract more value from customers who may be better off under the standard service compared to the premium service. Second, a small market size makes it difficult to justify investment in capacity in the standard service, which requires a large market to achieve economies of scale. In particular, while Boyaci and Ray (2003) find inverse relationship between service differentiation and unit capacity cost difference \((c_1 - c_2)\) under pay-per-use pricing and no market segmentation, we find the opposite, which may be due to differences in demand and pricing models. For example, in Lemma 2, the market size does not impact the optimal pricing, implying that being a monopolist in cloud services is not as attractive as it appears.

Based on both Proposition 3(a) and 3(b), we find that higher minimum usage \((m_0)\), lower usage heterogeneity \((b)\), and unit capacity cost difference \((c_1 - c_2)\) reduce the market size threshold \((\hat{a})\) and make it more attractive for both the service provider and the customers to have two classes of service. Moreover, it is interesting to note that neither the incremental value of service \((k)\) nor the fixed access utility \((v_0)\) impacts this choice of service offering, since the market coverage effect applies only to the premium service and not to the standard service. In sum, although service differentiation benefits consumers, it is not necessarily in the interests of the monopolist provider to do so, and therefore some policy enforcement may be needed.

4 Duopoly Scenarios

Similar to the extant literature in capacitated competition (e.g., Wang et al. 2016), we now consider duopoly settings to capture the effect of competition among service providers.
4.1 A Single Service Class for Each Provider

As discussed in Section 1.1, service differentiation may enhance the value proposition of a provider. As an example, Adobe may find a competitor contemplating to provide a cloud service similar to its Creative Cloud with lower QoS. Hence, in this section, we consider a scenario where the incumbent provider offers only the premium (class-1) service, while the entrant provider offers only the standard (class-3) service.

4.1.1 Use A Single Service Class to Deter the Entrant: Under this scenario, we analyze how the incumbent provider uses the premium service to deter the entrant provider from market entry. The indifference point (in delay sensitivity) between class-1 and class-3 service is \( \theta_4 = \frac{(p_1 - p_3)}{(d_3 - d_1)m} \). Hence, the number of users of the class-3 service is:

\[
N_3 = \int_{m_0}^{+\infty} [f(m) \cdot \theta_4] dm, \quad (3)
\]

and the total usage is:

\[
U_3 = \int_{m_0}^{+\infty} [mf(m) \cdot \theta_4] dm. \quad (4)
\]

Both \( N_3 \) and \( U_3 \) are functions of \( p_1 \). Using backwards induction, we analyze the entrant provider first. The entrant firm’s profit maximization problem can now be written as:

\[
\max_{(p_3, u_3)} \Pi_{e,a} = N_3 \cdot p_3 - e_2 \cdot q \cdot u_3;
\]

\[
s.t. \ u_3 \geq U_3 + 1/d_3.
\]

The incumbent provider attempts to ensure that the best possible profit of the entrant firm is zero, i.e., \( \max_{(p_3, u_3)} \Pi_{e,a} = 0 \).

**Lemma 3.** Under the duopolist scenario with a single service class from each provider, if the incumbent provider uses the Deter strategy,

(a) The optimal pricing is \( p_{1a}^d = \frac{2\sqrt{ab(b+1)(d_2-d_1)d_2e_2m_0q + (b+1)d_2e_2m_0q}}{ab_2d_2} \).

(b) The optimal capacity level is \( u_{1a}^d = \frac{1}{(b-1)b_2d_2} \left\{ ad_2e_2m_0q + 2\sqrt{b} + 1\sqrt{ad_2(-d_1 + d_2)e_2m_0q} - 2b^{3/2}\sqrt{b} - 1\sqrt{ad_2(d_2 - d_1)e_2m_0q} - b(d_2 + ad_2v_0) + b^2d_2[1 + a(km_0 - e_2m_0q + v_0)] \right\} \).
(c) This solution is a subgame perfect Nash equilibrium. Furthermore, this is a unique equilibrium as long as none of the players has the same payoffs at two terminal nodes.

In a realistic scenario, it is unlikely to have the same payoff at any two terminal nodes for any of the players. Hence, the result presented in Lemma 3 is a unique subgame perfect Nash equilibrium (Mas-Colell et al. 1995, p. 276). All the equilibrium solutions presented in this paper have the same property. We do not state it explicitly henceforth for brevity.

4.1.2 Use A Single Service Class but Allow Entry: An example similar to this scenario is that despite taking the market lead in stock-photo cloud service with its iStock offering, Shutterstock did not deter Adobe’s entry into this market with a lower-price service (Roy and Maan 2015), demonstrating that a possible competitive strategy is allowing competitor entry. We now analyze the equilibrium decisions and profit of the incumbent provider under this strategy.

Using backwards induction, we analyze the entrant provider first. Using \( N_3 \) and \( U_3 \) from Equations (3) and (4) respectively, the entrant firm’s profit maximization problem can now be written as:

\[
\max \Pi_{e,a} = N_3 \cdot p_3 - e_2 \cdot q \cdot u_3;
\]

\[
s.t. \ u_3 \geq U_3 + 1/d_3.
\]

Recall that the indifference point (in delay sensitivity) between class-1 and class-3 service is \( \theta_4 = \frac{(p_1 - p_3)}{(d_3 - d_1)} m \). Hence, the number of subscribers of the premium service is:

\[
N_{1p} = \int_{m_0}^{+\infty} [f(m) \cdot (\theta_1 - \theta_4)] dm,
\] (5)

and the total usage of the premium service is:

\[
U_{1p} = \int_{m_0}^{+\infty} [mf(m) \cdot (\theta_1 - \theta_4)] dm.
\] (6)

The incumbent firm’s profit maximization problem can now be written as:

\[
\max \Pi_{i,a} = N_{1p} \cdot p_1 - e_1 \cdot u_1;
\]

\[
s.t. \ u_1 \geq U_{1p} + 1/d_1.
\]
Lemma 4. Under the duopolist scenario with a single service class each and using the Allow strategy, the equilibrium prices are:

(a) \( p_{1a}^{nd} = \frac{-(1+b)m_0(2d_2(e_1+k)+d_1(-2k+e_2q))+2b(d_1-d_2)\nu_0}{6(d_1-4d_2)} \) for the incumbent provider.

(b) \( p_{3a}^{nd} = \frac{-(1+b)m_0(-d_1k+d_2(e_1+k+2e_2q))+b(d_1-d_2)\nu_0}{6(d_1-4d_2)} \) for the entrant provider.

The expressions of equilibrium capacity levels and profits are very tedious, and thus shown in the Online Appendix for brevity.

4.2 Multiple Classes of Service

In this section, we now consider a duopoly scenario where the incumbent provider offers both the premium (class-1) service and the standard (class-2) service, while the entrant provider offers a standard (class-3) service only, which is identical to the incumbent provider’s standard (class-2) service except the price. We investigate when and why the incumbent provider may decide to deter or allow competitor entry.

4.2.1 Provide Both Premium and Standard Services to Deter Entry: Under this strategy, despite the additional flexibility of using the standard service to deter entry, the incumbent firm needs to build capacity in the standard service and be ready to serve the customers in this segment. By doing so, the incumbent firm can actually limit the price of competitor’s standard (class-3) service using the incumbent provider’s own standard (class-2) service, which forces the entrant to earn zero profit instead of leaving room for the entrant to manipulate their pricing \( (p_3) \).

The downside of this strategy is that some customers using the standard service may be otherwise profitable under the premium service, and thus cannibalization may occur.

Using backwards induction, we analyze the entrant provider first. Using \( N_3 \) and \( U_3 \) from Equations (3) and (4) respectively, the entrant firm’s profit maximization problem can be written as:

\[
\max_{(p_3, u_3)} \Pi_{e,b} = N_3 \cdot p_3 - e_2 \cdot q \cdot u_3;
\]

s.t. \( u_3 \geq U_3 + 1/d_3 \).
The optimal \( p_3 \) is a function of \( p_1 \) and \( p_2 \), and the incumbent provider attempts to find \( p_1, p_2, u_1, \) and \( u_2 \) such that the best possible profit of the entrant firm is zero (i.e., \( \max_{(p_3,u_3)} \Pi_{e,b} = 0 \)) while still maximizing the incumbent provider’s own profit. Using \( N_2 \) and \( U_2 \) from Equations (1) and (2), respectively; and \( N_{1p} \) and \( U_{1p} \) from Equations (5) and (6), respectively, the incumbent firm’s profit maximization problem can be written as:

\[
\max_{(p_1,p_2,u_1,u_2)} \Pi_{i,b} = N_{1p} \cdot p_1 + N_2 \cdot p_2 - e_1 \cdot u_1 - e_2 \cdot u_2;
\]

\[
s.t. \quad u_1 \geq U_{1p} + 1/d_1;
\]

\[
u_2 \geq U_2 + 1/d_2;
\]

\[
\max_{(p_3,u_3)} \Pi_{e,b} = 0.
\]

We can obtain the equilibrium pricing under this strategy, but the expressions are very complicated and thus shown in the Online Appendix for brevity. For ease of exposition, we list in Lemma 5 the equilibrium prices when \( e_1 = e_2 \) and \( v_0 = 0 \):

**Lemma 5.** Under the scenario that the incumbent provider offers both premium and standard services and use the Deter strategy, the price of the premium (class-1) service is:

\[
p_{1b}^d = \frac{(b+1)m_0[(2d_2 - d_1)(e_1 + k) + 2d_1e_1q]}{4bd_2} - \frac{d_1 \sqrt{a(1+b)m_0(a(1+b)m_0(e_1 + k - 2e_1q)^2 - 16be_1q)}}{4abd_2},
\]

and the price of the standard (class-2) service is:

\[
p_{2b}^d = \frac{b+1}{8b(d_2 - d_1)} \left\{ m_0(k + e_1)(3d_1^2 - 5d_1d_2 + 2d_2^2) + m_0q(2d_1^2e_1 - 6d_1d_2e_1 + 4d_2^2e_1) \right\} + \frac{d_1}{8b(d_2 - d_1)} \sqrt{(b+1)m_0[a(1+b)m_0(e_1 + k - 2e_1q)^2 - 16be_1q]} - \frac{1}{8b(d_2 - d_1)} \sqrt{\frac{2m_0(b+1)}{d_2 - d_1}} \cdot \left\{ 8b(d_1 - 2d_2)^2(d_1 - d_2)e_1q - a(b+1)(d_1^2 - 3d_1^2d_2 + 4d_1 - 2d_2^2 - 2d_1^2)m_0(e_1 + k - 2e_1q)^2 + \sqrt{a}(d_1 - 2d_2)(e_1 + k - 2e_1q)(d_2 - d_1) \sqrt{m_0(a(1+b)m_0(e_1 + k - 2e_1q)^2 - 16be_1q)} \right\}^{1/2}.
\]

**4.2.2 Provide Both Premium and Standard Services but Allow Entry:** Since the entry is allowed for the entrant provider in this strategy, the entrant will need to undercut the price of class-2 service with \( p_3 < p_2 \), which leads to no users for the incumbent provider’s standard (class-2) service. Therefore, \( p_2 \) is arbitrary as long as \( p_2 > \max\{p_{2b}, p_3\} \).
Using backwards induction, we analyze the entrant provider first. Using $N_3$ and $U_3$ from Equations (3) and (4) respectively, the entrant firm’s profit maximization problem can be written as:

$$\max_{(p_3,u_3)} \Pi_{e,b} = N_3 \cdot p_3 - e_2 \cdot q \cdot u_3;$$

$$s.t. \ u_3 \geq U_3 + 1/d_3.$$

Using $N_2$ and $U_2$ from Equations (1) and (2), respectively; and $N_{1p}$ and $U_{1p}$ from Equations (5) and (6), respectively, the incumbent firm’s profit maximization problem can be written as:

$$\max_{(p_1,p_2,u_1,u_2)} \Pi_{i,b} = N_{1p} \cdot p_1 + N_2 \cdot p_2 - e_1 \cdot u_1 - e_2 \cdot u_2;$$

$$s.t. \ u_1 \geq U_{1p} + 1/d_1;$$

$$u_2 \geq U_2 + 1/d_2.$$

The equilibrium solutions are given in Lemma 6.

**Lemma 6.** Under the scenario that the incumbent provider offers both premium and standard services and use the Allow strategy, the equilibrium prices are:

(a) $p^{nd}_{1b} = \frac{-(1+b)m_0(2d_2(k_1+k)+d_1(-2k+e_2q)) + 2b(d_1-d_2)v_0}{6(d_1-4d_2)}$ for class-1 service.

(b) $p^{nd}_{3b} = \frac{-(1+b)m_0(-d_1k+d_2(e_1+k+2e_2q)) + b(d_1-d_2)v_0}{6(d_1-4d_2)}$ for class-3 service.

The equilibrium capacity levels and profits are provided in the Online Appendix. It is worth noting that there is no incentive for the incumbent provider to undercut the price of the entrant’s standard service, since lower standard service pricing will hurt the profitability of the premium service that the incumbent runs. Also, note that under the Allow Strategy, the incumbent provider chooses to offer only the premium service due to the extra cost of setting up minimum capacity for the standard service that no user chooses.

### 4.3 Results and Managerial Insights

In this subsection, we report results on competitive strategies and discuss managerial insights.
4.3.1 Impact of Market Size: We first focus on how the market size impacts the incumbent provider’s profit, when a single class of service is employed to deter the entrant.

Proposition 4. When \( m_0(1+b)(e_1 + k - 2e_2q) + bv_0 > 0 \), the equilibrium profit of the incumbent provider under the Premium-only Deter strategy decreases in market size when the market size is above a threshold \( a_c \), and vice versa, where

\[
a_c = \frac{(b-1)^2be_2q(d_2 - d_1)((b+1)m_0(e_1 - 2e_2q + k) + bv_0)^2}{(b+1)d_2m_0(b^2(e_1 - e_2q)((-e_2m_0q + km_0 + v_0) + bv_0(e_2q - e_1) + e_2m_0q(e_1 - e_2q + k))^2}.
\]

The condition \( m_0(1+b)(e_1 + k - 2e_2q) + bv_0 > 0 \) is easily satisfied when the capacity cost differential \( q \) is not extremely high. Interestingly, according to Proposition 4, when the market size grows, the profit may not follow. In a larger market, there is stronger incentive for competitor entry, and then the incumbent provider responds with lower pricing, which in turn attracts more heavy users who are delay-sensitive. Therefore, a larger market size may boost revenue despite lower pricing, and higher usage per subscriber leads to higher capacity cost per subscriber.

When the market size is low (below \( a_c \)), the increase in revenue outweighs the increase in capacity cost, and hence the equilibrium profit continues to increase in market size; when the market size is high (above \( a_c \)), the increase in capacity cost and the decrease in pricing outweigh the increase in revenue. Thus, it becomes unprofitable for the incumbent firm to stick to the Premium-only Deter strategy as the market size grows. Therefore, managers need to acknowledge this change in competitive incentives as the market size evolves, and need to be prudent in choosing their strategies. We will further discuss the impact of market size in Proposition 5(c) and Observation 2.

4.3.2 Deter or Allow? In this subsection, we use the findings of Section 4.1 (single class of service for each provider) to answer the following question: Should the incumbent provider deter or allow competitor entry? While new entrant threat is detrimental for the incumbent provider, Deter strategy should not always be adopted, especially when the entrant provider targets at the low-end customers, such as Salesforce’s entry in wealth management services dominated by Bloomberg (THINKstrategies 2007). In this subsection, we investigate the Deter/Allow decision of
the incumbent firm and analyze factors influencing this strategy choice. We begin by presenting
the profit difference between these two strategies denoted as $\Delta \Pi_{1a}^{d-nd} = \Pi_{1a}^d - \Pi_{1a}^{nd}$. We analyze the
sign of $\Delta \Pi_{1a}^{d-nd}$ to identify the best strategy of the incumbent provider in Proposition 5 under
assumptions $e_1 = e_2$ and $v_0 = 0$. Our extensive experiments show that the qualitative results and key
insights do not change for a general case with unequal unit capacity cost and non-zero access utility.

**Proposition 5.** We have the following results regarding Deter/Allow strategy choice:

(a) The Deter strategy outperforms the Allow strategy when the customer valuation of service (i.e.,
$k$) is within an interval $(k_1, k_2)$, and vice versa.

(b) The Deter strategy outperforms the Allow strategy when the minimum customer usage (i.e.,
$m_0$) is within an interval $(m_1, m_2)$, and vice versa.

(c) The Deter strategy outperforms the Allow strategy when the market size (i.e., $a$) is within an
interval $(a_1, a_2)$, and vice versa.

Note that $\{k_1, k_2\}, \{m_1, m_2\}$, and $\{a_1, a_2\}$ are roots of second-order polynomial equations shown
in the proof of Proposition 5. Since the pricing under the Deter strategy is independent of $k$ (see
Lemma 3), higher $k$ leads to more heavy-usage subscribers and drives the marginal profit per user
lower due to increased capacity cost and unchanged revenue per user. Under the Allow strategy, as
$k$ increases, the pricing of the incumbent provider increases faster than that of the entrant provider.
Thus, some heavy users may become subscribers (but less so compared to that under the Deter
strategy), and the entrant may have a larger market share due to the increased price difference. It
follows that when customer valuation of service is very high, the incumbent provider should not
deter the entrant provider due to higher pricing and relatively low usage of the premium service
under the Allow strategy. When customer valuation of the service is very low, Allow strategy
prevails since entry incentive is low. Under moderate customer valuation of the service, i.e., $k \in
(k_1, k_2)$, the Deter strategy is more desirable since the pricing under the Allow strategy is still
too low to justify losing low-usage, low-delay-sensitivity subscribers to the entrant provider. This
result aligns with the finding of Dafny (2005) that incumbent hospitals in moderately-attractive
markets generate the strongest volume growth following an increase in reimbursement (likely for entry-deterrence purposes), greater than those in unattractive or very attractive markets. This result also indicates that the managers need to be careful in choosing the best competitive strategy and balance between wider customer base and higher capacity cost.

Compared to the customer valuation of the service, the minimum usage relates directly to customer utility and capacity usage. We find that the pricing of incumbent firm under the Deter strategy increases in the minimum usage with diminishing returns, while pricing under the Allow strategy increases linearly in the minimum usage. It follows that when the minimum usage is very large, deterring entry entails relatively low pricing that attracts heavy-usage customers, which hurts the revenue more than the revenue lost to the competitor under the Allow strategy. When the minimum usage is small, allowing competitor entry may not hurt the profit much, since customer utility gained from service usage is low and it is difficult for the competitor to make a profit in this market. When the minimum usage is moderate, the incumbent firm may benefit from a large subscription base by deterring competitor entry, and avoid being hurt too much by customer usage.

Proposition 5(c) relates to Proposition 4 since larger market size may reduce the profit under the Deter strategy, but always increases the profit under the Allow strategy. Also, note that the pricing under the Deter strategy decreases in market size, but pricing under the Allow strategy does not depend on market size. It follows that when the market size is very large, it can be very costly to deter competitor entry with low pricing and thus the Allow strategy is better. When the market size is very small, Allow strategy is preferred since it is difficult for the entrant to make a profit in a small market. When the market size is moderate, the incumbent firm chooses the Deter strategy since benefits of dominating the market outweighs price discount needed to deter the entrant. The managers of the incumbent firm need to realize that, in a larger market, the strategy focus should shift from deterrence to containment. We speculate that Shutterstock did not deter competitor entry since the market size for photo services is large, which agrees with both Proposition 4 and Proposition 5(c).

Another implication of the Deter/Allow choice lies in consumer surplus. As demonstrated in Figure 2, the Deter strategy creates more consumer surplus than the Allow strategy in an attractive
market with a large customer base and high customer valuation of the service. It seems counter-intuitive that maintaining monopoly in an attractive market creates higher consumer surplus than a duopoly. However, given the difficulty involved in Deter strategy (as shown in Proposition 4), it is understandable that the Deter strategy involves lower pricing and benefits consumers, thus should not be discouraged by the policy-makers.

![Figure 2](image)

Figure 2 Consumer surplus difference (Allow - Deter) ($d_1 = 1, d_2 = d_3 = 2, e_1 = 0.7, e_2 = 0.7, m_0 = 0.8, b = 2, v_0 = 0, q = 1$)

4.3.3 Competitive Strategy and Service Differentiation: In this subsection, we use the findings of both Sections 4.1 (single class of service for each provider) and 4.2 (multiple classes of service for the incumbent firm) to explore the interplay between competitive strategy and service offering strategy. We compare three strategies: (1) Premium-Only Deter, (2) Dual-Class Deter, and (3) Premium-Only Allow. The analytic expressions in this subsection are too cumbersome, and therefore we use numerical analysis. We focus on the incumbent provider’s strategy choice under the impact of various parameters and how the Deter/Allow decision and the service differentiation decision intertwine.
In Figure 3, we plot the optimal strategy of the incumbent provider with respect to the customer valuation of service and the (standard service) capacity cost differential between providers. We find that the Deter strategies are preferred when the capacity cost differential is high and the customer valuation of service is low. The intuition is that high customer valuation of service (a proxy for service value) encourages market-entry and makes deterrence costly, similar to what we find in Propositions 4 and 5(c). Moreover, when the capacity cost differential (which is a proxy for cost leadership in Porter (1980)) is high, the entrant provider is at a more disadvantageous position, and thus Deter strategies become more desirable.

The incumbent firm’s strategy should adapt to both service value and cost leadership. Managers should be prepared to adopt the Premium-Only Allow strategy if the incumbent firm is in a market with high service value and minimal cost leadership, as is the case for Spotify in the music-streaming market. However, for a stable market where service value is usually low to moderate, the incumbent firm is advised to maintain deterring potential entrants by adding the standard service when the customer valuation increases or the incumbent provider’s cost leadership diminishes. The standard service can be a strategic weapon whose usefulness is amplified by cost leadership while also contributing to service differentiation. For the entrant provider, the focus should be a high-value market with minimal cost differential. Here, we summarize our findings in Observation 1.

**Observation 1**  
*Deter strategies work best when the capacity cost differential is high and the customer valuation of service is low.*

To investigate how the optimal strategy may adapt to market characteristics, in Figure 4, we plot the optimal strategy of the incumbent provider with respect to the customer valuation of service and the market size. We find that Deter strategies work best when the market size is relatively small with low customer valuation of service. The intuition is that both a larger market and a higher customer valuation of the service encourage competitor entry and make it more costly to deter the entrant. We also find that the Dual-Class Deter strategy is a preferred strategy when the market size is large enough to justify service differentiation.
On the one hand, the managers need to beware that introducing the standard service is unlikely to help if they happen to operate in a small market with high customer valuation, as is the case in many niche markets. On the other hand, the managers should leverage on the standard service when the market size is large with moderate customer valuation in order to strengthen the competitive position of the incumbent provider against the entrant, as is the case for many online storage solutions. Sticking to Premium-only Deter in a large market does no good. The entrant provider should focus on a large high-value market to avoid entry-deterrence strategies. Here, we summarize these findings in Observation 2.

**Observation 2** Deter strategies work best when the market size is relatively small and the customer valuation of service is low. Moreover, the Dual-Class Deter strategy outperforms the Premium-only Deter strategy when the market size is relatively large.

To investigate how the optimal strategy may adapt to capacity cost characteristics, in Figure 5, we plot the optimal strategy of the incumbent provider with respect to the unit capacity cost and the standard service capacity cost differential. We find that Deter strategies work best when both
the incumbent provider’s unit capacity cost and the standard service cost differential are high. The intuition is that a higher unit capacity cost results in lower margin for both providers, and thus the Deter strategies may lower the price only slightly but can still force the entrant provider out of the market.

Moreover, as the incumbent provider’s capacity cost decreases, it should maintain a Deter strategy, although adding the standard service is increasingly helpful. However, it is difficult to deter potential entrants when the cost leadership is weak (i.e., capacity cost differential is low), and this could be why incumbent providers of common services (e.g., music/video streaming services) seldom try to deter entrants. The managers should acknowledge this phenomenon and choose their strategies wisely. Here, we summarize the findings in Observation 3.

Observation 3 Deter strategies are preferred when both the incumbent provider’s unit capacity cost and the standard-service cost differential are high.

5 Conclusions and Future Research Directions

Our paper considers strategic decisions of cloud service providers to deter or allow competitor entry under subscription pricing. We adopt a game-theoretic framework to investigate how the cloud ser-
vice providers should respond to competitive threats via pricing and capacity decisions. We analyze situations where the incumbent firm provides single/dual-class(es) of service, and we explore the impact of capacity cost differential. Understanding the intricacies caused by subscription pricing is crucial to effective management of cloud services since higher customer usage demands higher capacity levels but may not lead to higher profit as is the case in fee-for-service business models.

Our analyses provide several useful managerial insights. First, we find that the usage distribution of potential subscribers may impact strategy choice of the incumbent cloud service provider. In particular, we find that the higher usage is not always detrimental to a provider’s profit and may provoke changes in competitive strategy. Second, we find that providing sufficient service value to the customers is critical to the success of a cloud service provider and may moderate how usage impacts profit. The high value of the service may encourage potential entrants to enter the market, which triggers the incumbent provider to switch from the Deter strategy to the Allow strategy in a not-too-small market. Third, we find that the market size influences strategy choice and service differentiation. Generally, a larger market encourages service differentiation that benefits consumers in a monopolist market. However, in a competitive market, larger market size generally entails

Figure 5  Strategy choice \( (d_1 = 1, d_2 = d_3 = 2, k = 3, m_0 = 3, b = 3, a = 10) \)
more competition. Higher competition reduces profit under the Deter strategy and encourages the incumbent provider to choose the Allow strategy over the Deter strategy, which, interestingly, does not necessarily benefit consumers. Fourth, we demonstrate that while entrant threat is detrimental to the incumbent provider, the Deter strategy is not always the best. Concurring with Mark Organ, the CEO and founder of Influitive, we find that the incumbent provider may want to leverage on its unique strength to dominate market segments and deliver to the under-served customers (Lemkin 2015), possibly via service differentiation. Lastly, we discover that the capacity cost, especially the cost differential of the standard service, may significantly impact the strategy choices. Specifically, the low standard-service cost differential and the low incumbent capacity cost favor the Allow strategy to leverage on the competitiveness of premium service, while the high standard-service cost differential and the high incumbent capacity cost favor the Deter strategy to leverage on the efficiency of standard service.

Our work is by no means an exhaustive study of cloud service competition, but rather an initial attempt to understand how subscription pricing impacts operational decisions and the competitive strategy choice in the context of cloud service. Future work may include market entry strategies of potential entrants of a cloud service; it would be interesting to know what traits are needed for successful market entries where physical distances are no longer entry barriers. Our work also opens new dimensions to applying operations management frameworks to services where customer usage may drive up cost but do not contribute to revenue in the same manner, which creates incentives for service providers to manage customer usage via pricing and capacity decisions.

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Note: All online references in the following list are last accessed on November 18, 2016.
References


Electronic Companion

Online Appendix to
Should You Kill or Embrace Your Competitor: Cloud Service and Competition Strategy

**Proof of Lemma 1**

It is easy to show that the optimal capacity satisfies

\[ u_1 = U_1 + 1/d_1. \]  \hspace{1cm} (EC.1)

The optimal \( p_1 \) can be obtained as

\[ p_{1a} = \frac{m_0(1 + b)(e_1 + k) + bv_0}{2b} \]  \hspace{1cm} (EC.2)

using the first-order condition \( \frac{\partial \Pi_1}{\partial p_1} = 0 \). The second-order condition is satisfied since \( \frac{\partial^2 \Pi}{\partial p_1^2} = \frac{2ab}{d_1 m_0(b+1)} < 0 \). Substituting Equation (EC.2) into Equation (EC.1), we arrive at the expressions of \( u_{1a} \) and \( \Pi_a \) in Lemma 1. \( \square \)

**Proof of Lemma 2**

It is easy to show that the optimal capacity decisions satisfy

\[ u_1 = U_{1b} + 1/d_1; \] \hspace{1cm} (EC.3)
\[ u_2 = U_2 + 1/d_2. \] \hspace{1cm} (EC.4)

Based on the first-order conditions \( \frac{\partial \Pi}{\partial p_{1b}} = 0 \) and \( \frac{\partial \Pi}{\partial p_{2b}} = 0 \), the optimal prices are

\[ p_{1b} = \frac{m_0(1 + b)(e_1 + k) + bv_0}{2b} \] \hspace{1cm} (EC.5)
\[ p_{2b} = \frac{m_0(1 + b)(e_2 + k) + bv_0}{2b}. \] \hspace{1cm} (EC.6)
We verify the second-order condition by analyzing the Hessian matrix

\[
H = \begin{bmatrix}
\frac{2abd_2}{(1+b)d_1(d_1-d_2)m_0} & \frac{2ab}{(1+b)(d_1-d_2)m_0} \\
\frac{2ab}{(1+b)(d_1-d_2)m_0} & \frac{2ab}{(1+b)(d_1-d_2)m_0}
\end{bmatrix}
\]

The first-order leading principal minor is negative. The second-order leading principal minor is \(\det(H) = \frac{4a^2b^2}{(1+b)^2d_1(d_2-d_1)m_0^2} > 0\). Therefore, \(H\) is negative-definite, and thus the second-order condition is satisfied. Substituting Equations (EC.5) and (EC.6) into Equations (EC.3) and (EC.4), we arrive at the optimal capacity decisions listed in Lemma 2. We also have the optimal profit:

\[
\Pi_b = \frac{1}{4b(-1+b^2)d_1(d_1-d_2)d_2m_0} \{ ad_2(d_2(e_1+k)^2 - d_1(2e_1-e_2+k)(e_2+k))m_0^2 \\
+ b^3(m_0(-4(d_1-d_2)(d_2e_1+d_1e_2) - ad_2(d_2(e_1-k)^2 + d_1(-2e_1e_2 + e_2^2 + 2e_1k-k^2))m_0) \\
- 2a(d_1-d_2)d_2(e_1-k)m_0v_0 + a(d_1-d_2)d_2v_0^2 \\
+ ab^2d_2(-d_2(e_1-k)^2 + d_1(-2e_1e_2 + e_2^2 + 2e_1k-k^2))m_0^2 + (-d_1+d_2)v_0^2 \\
+ bm_0(4d_1^2e_2 + d_2^2((ae_1m_0 + ak(km_0+2v_0) + 2e_1(-2+akm_0-av_0)) \\
- d_1d_2(4e_2 - ae_2^2m_0 + ak(km_0+2v_0) + 2e_1(-2+a(e_2+k)m_0-av_0)))}\}
\]

\[\square\]

**Proof of Proposition 1**

We have that

\[
\frac{d\Pi_a}{dm_0} = \frac{a(b+1)m_0^2(be_1-bk+e_1+k)((b-1)e_1-(b+1)k) - a(b-1)b^2v_0^2}{4b(b^2-1)d_1m_0^2},
\]

and it can be shown that when \(v_0 = 0\),

\[
\frac{d\Pi_a}{dm_0} = \frac{a(be_1-bk+e_1+k)((b-1)e_1-(b+1)k)}{4b(b-1)d_1}
\]

Setting \(\frac{d\Pi_a}{dm_0} = 0\), we have a positive solution

\[
\hat{m}_0 = \frac{\sqrt{b-1}bv_0}{\sqrt{(b+1)(be_1-bk+e_1+k)((b-1)e_1-(b+1)k)}},
\]

beyond which \(\frac{d\Pi_a}{dm_0} > 0\). \(\hat{m}_0\) does not exist when \(k < \frac{b+1}{b-1}e_1\) or equivalently, \(be_1-bk+e_1+k > 0\). \[\square\]
Proof of Proposition 2

Note that
\[
\frac{d\Pi_b}{db} = \frac{-a((b - 1)^2e_2^2 - 2(-1 + b(2 + b))e_2k + (b - 1)^2k^2)m_0}{4(-1 + b)^2b^2d_1}.
\]
Solving \( \frac{d\Pi_b}{db} = 0 \) yields two roots of \( b \). The smaller root of the two is
\[
\hat{b} = \frac{e_2^2 + 2e_2k + k^2 - 2(e_2 + k)\sqrt{e_2k}}{(e_2 - k)^2} = \frac{k + e_2}{(\sqrt{k} + \sqrt{e_2})^2} < 1,
\]
thus not feasible. The larger root of the two is
\[
\hat{b} = \frac{e_2^2 + 2e_2k + k^2 + 2(e_2 + k)\sqrt{e_2k}}{(e_2 - k)^2}.
\]
Since
\[
\lim_{b \to +\infty} \frac{d\Pi_b}{db} = -\frac{a(e_2 - k)^2m_0}{4d_1} < 0,
\]
and
\[
\frac{\partial \Pi_b}{\partial b} \bigg|_{b=2} = -\frac{am_0(e_2^2 - 14e_2k + k^2)}{16d_1} = -\frac{am_0[k - (7 - 4\sqrt{3})e_2][k - (7 + 4\sqrt{3})e_2]}{16d_1},
\]
which is positive whenever \((7 - 4\sqrt{3})e_2 < k < (7 + 4\sqrt{3})e_2\). Clearly, the unit capacity cost of class-2 service (i.e., \( e_2 \)) cannot be more than the incremental utility for each unit of usage (i.e., \( k \)). Therefore, \((7 - 4\sqrt{3})e_2 < k\) is always satisfied. Hence, whenever \( k < (7 + 4\sqrt{3})e_2\), we conclude that \( \frac{d\Pi_b}{db} < 0 \) when \( b > \hat{b} \) and vice versa. \( \square \)

Proof of Proposition 3

It follows from
\[
\Pi_b - \Pi_a = \frac{am_0(b + 1)(e_1 - e_2)^2}{4b} - \frac{e_2}{d_2}
\]
that \( \Pi_b > \Pi_a \) when \( a > \hat{a} = \frac{4be_2}{m_0d_2(b + 1)(e_1 - e_2)^2} \), which does not depend on \( k \) and \( v_0 \).

Recall that \( N_1 = \int_{m_0}^{+\infty} \left[ f(m) \cdot \frac{\theta_0(p_{1a}) - 0}{(-1 - 0)} \right] dm \) under monopoly and single-class service is a function of \( p_1 \), where \( \theta_1 \) is a function of \( p_1 \). We obtain a function \( p_{1a}^f(\cdot) = N_1^{-1}(p_{1a}) \). Let \( N_{1a} \) be the optimal number of users under monopoly and single-class service. We can calculate the consumer surplus under monopoly and premium service only as
\[
C_{1a} = \int_0^{N_{1a}} [p_{1a}^f(n) - p_{1a}]dn = \frac{a[(1 + b)(e_1 - k)m_0 - bv_0]^2}{8b(1 + b)d_1m_0}.
\]
Recall that \( N_1 = \int_{m_0}^{+\infty} \left[ f(m) \cdot \frac{\theta_1 - \theta_3}{m} \right] dm \) and \( N_2 = \int_{m_0}^{+\infty} \left[ f(m) \cdot \frac{\theta_3 - 0}{m} \right] dm \) under monopoly and dual-class service. Note that \( \theta_1 \) is a function of \( p_1 \), and \( \theta_3 \) is a function of \( p_1 \) and \( p_2 \). Solving the set of equations below

\[
\begin{cases}
N_1(p_1, p_2) = n_1 \\
N_2(p_1, p_2) = n_2
\end{cases}
\]

we obtain two functions \( p_{1b}(\cdot, \cdot) \) and \( p_{2b}(\cdot, \cdot) \).

\[
\begin{cases}
p_1 = p_{1b}(n_1, n_2) \\
p_2 = p_{2b}(n_1, n_2)
\end{cases}
\]

Let \( N_{1b} \) and \( N_{2b} \) be the optimal number of users under monopoly and dual-class service. Using the technique presented in Pressman (1970), we can calculate the consumer surplus under monopoly and dual-class service as:

\[
C_{1b} = \int_{(0,0)}^{(N_{1b}, N_{2b})} \left\{ [p_{1b}(n_1, n_2) - p_{1b}]dn_1 + [p_{2b}(n_1, n_2) - p_{2b}]dn_2 \right\}
\]

\[
= \int_0^{N_{1b}} [p_{1b}(n_1, 0) - p_{1b}]dn_1 + \int_0^{N_{2b}} [p_{2b}(n_1, n_2) - p_{2b}]dn_2
\]

\[
= \frac{a(d_1[(1 + b)(e_2 - k)m_0 - bv_0)]2(1 + b)e_1m_0 - (1 + b)(e_2 + k)m_0 - bv_0)}{8b(1 + b)d_1(d_1 - d_2)m_0}
\]

\[
+ \frac{ad_2(1 + b)(e_1 - k)m_0 - bv_0)^2}{8b(1 + b)d_1(d_2 - d_1)m_0}.
\]

It follows that the incremental consumer surplus of introducing the standard service is

\[
C_{1b} - C_{1a} = \frac{a(1 + b)(e_1 - e_2)^2m_0}{8b(d_2 - d_1)},
\]

which increases quadratically in the unit capacity cost difference, and decreases in the delay guarantee difference.

\[\square\]

PROOF OF LEMMA 3

It is easy to show that the optimal capacity decision satisfy \( u_3 = U_3 + 1/d_3 \). Note that \( \Pi_{e,a} \) is a parabolic function of \( p_3 \) with \( \frac{d^2\Pi_{e,a}}{dp_3^2} = \frac{2ab}{m_0(k+1)(d_1-d_2)} < 0 \), satisfying the second-order condition. The optimal \( p_3 \) is a function of \( p_1 \):

\[
p_3^* = \frac{1}{2}p_1 + \frac{(1 + b)e_2m_0q}{2b}.
\]

(EC.7)
We have the value of $p_1$ such that the best possible profit of the entrant firm $\Pi^*_e,a = 0$:

$$p^*_1 = \frac{2\sqrt{ab(b+1)(d_2 - d_1)d_2 e_2 m_0 q} + a(b+1)d_2 e_2 m_0 q}{ab^2 d_2}.$$

It follows that the capacity level is $u^*_1$ listed in Lemma 3 and the profit of the incumbent firm is

$$\Pi^*_i,a = \frac{1}{b^3(b^2 - 1)d_1 d_2^2 m_0} \left\{ b(b+1) d_2 e_1 m_0 \left[2b\sqrt{ab^3(b+1)d_2 e_2 m_0 q(d_2 - d_1)} - abd_2 e_2 m_0 qight. \\ - 2\sqrt{ab^3(b+1)d_2 e_2 m_0 q(d_2 - d_1)} - d_2 b^3(a(-e_2 m_0 q + km_0 + v_0) + 1) + b^2(ad_2 v_0 + d_2) \\ - (b - 1)(2\sqrt{ab^3(b+1)d_2 e_2 m_0 q(d_2 - d_1)} + ab(b+1)d_2 e_2 m_0 q) \right. \\ 2\sqrt{ab^3(b+1)d_2 e_2 m_0 q(d_2 - d_1)} - abd_2 (-b + 1)e_2 m_0 q + (b + 1)km_0 + bv_0) \left. \right\}.$$  

□

**Proof of Lemma 4**

Using backwards induction, we analyze the entrant provider first. The entrant firm’s profit maximization problem is:

$$\max_{(p_3,u_3)} \Pi_{e,a} = N_3 \cdot p_3 - e_2 \cdot q \cdot u_3;$$

$$s.t. u_3 \geq U_3 + 1/d_3.$$  

It is easy to show that the optimal capacity decision satisfy $u_3 = U_3 + 1/d_3$. The solution of this problem is $p^*_3 = \frac{bp_1 + (b+1)e_2 m_0 q}{2b}$. $\Pi_{e,a}$ is concave since $\frac{d^2\Pi_{e,a}}{dp_3^2} = \frac{2ab}{m_0(b+1)(d_2 - d_1)} < 0$.

The incumbent firm’s profit maximization problem is:

$$\max_{(p_1,u_1)} \Pi_{i,a} = N_{1p} \cdot p_1 - e_1 \cdot u_1;$$

$$s.t. u_1 \geq U_{1p} + 1/d_1.$$  

It is easy to show that the optimal capacity decision satisfy $u_1 = U_{1p} + 1/d_1$. The optimal $p_1$ in this case is a function of $p^*_3$:  

$$p^*_1 = \left(1 + b\right)d_3(e_1 + k)m_0 + bd_3 v_0 - d_1((1 + b)km_0 + b(v_0 - p^*_3))/2bd_3.$$  

(EC.8)
Note that $\Pi_{i,a|p_3=p_3^*}$ is concave in $p_1$ since $\frac{d^2\Pi_{i,a|p_3=p_3^*}}{dp_1^2} = -\frac{ab(2d_2-d_1)}{d_1m_0(b+1)(d_2-d_1)} < 0$. Combining Equation (EC.8) with Equation (EC.7), we can obtain the equilibrium prices:

\begin{align*}
p_{1a}^n &= \frac{-(1+b)m_0(2d_2(e_1+k) + d_1(-2k+e_2q)) + 2b(d_1-d_2)v_0}{b(d_1-4d_2)}.
\end{align*}

The equilibrium capacity levels are:

\begin{align*}
u_{1a}^n &= \frac{1}{(b-1)bd_1(d_1-4d_2)(d_1-d_2)} \left\{ am_0((d_1-2d_2)(d_1k-d_2(e_1+k)) - d_1d_2e_2q) \\
&\quad - b(d_1-d_2)(d_1-2d_2(2+av_0)) + b^2(d_1^2 + d_1d_2(-5 + am_0(e_1-2k+e_2q) - 2av_0) \\
&\quad + 2d_2^2(2+a(-e_1m_0 + km_0 + v_0)) \right\}.
\end{align*}

\begin{align*}
u_{3a}^n &= \frac{1}{bd_2(d_2-d_1)(4d_2-d_1)} \left\{ ad_2m_0(-d_1k + d_1d_2(e_1+k-2e_2q)) \\
&\quad + b(d_1^2 - d_1d_2(5+a(km_0 - e_2m_0q + v_0)) + d_2^2(4+a(m_0(e_1+k-2e_2q) + v_0)) \right\}.
\end{align*}

The equilibrium profits are:

\begin{align*}
\Pi_{i,a}^n &= \frac{1}{bd_1(d_1-4d_2)^2(d_1-d_2)} \left\{ \frac{1}{(1+b)m_0} \left[ -d_1((1+b)(e_1-2k)m_0 + e_2m_0q(b+1) - 2bv_0) \\
&\quad + 2d_2((1+b)e_1m_0 - (1+b)km_0 - bv_0)) \right] \right\} \\
&\quad \cdot \left\{ d_1(-2(1+b)k)(e_1 + (1+b)e_2m_0q - 2bv_0) \\
&\quad + 2d_2((1+b)e_1m_0 + (1+b)km_0 + bv_0) \right\} - \frac{1}{b-1}(d_1-4d_2)e_1 \left[ am_0(d_1^2k + 2d_2^2(e_1+k) \\
&\quad - d_1d_2(e_1+3k+e_2q)) - b(d_1-d_2)(d_1-2d_2(2+av_0)) \\
&\quad + b^2(d_1^2 + d_1d_2(-5+a(e_1m_0 - 2km_0 + e_2m_0q - 2v_0)) + 2d_2^2(2+a(-e_1m_0 + km_0 + v_0)) \right) \right\}.
\end{align*}

\begin{align*}
\Pi_{c,a}^n &= \frac{1}{4b(1+b)(d_2-d_1)d_2m_0} \left\{ (4b(1+b)(d_1-d_2)e_2m_0q + \frac{4ad_2}{(d_1-4d_2)^2} \right. \\
&\quad \cdot \left\{ [(1+b)m_0(-d_1k + d_1e_2q + d_2(e_1+k-2e_2q)) + b(d_2-d_1)v_0] \right\} \right\}.
\end{align*}

**Proof of Lemma 5**

Note that $N_2$ and $U_2$ are provided in Equations (1) and (2), respectively; $N_3$ and $U_3$ are provided in Equations (3) and (4), respectively; and $N_{1p}$ and $U_{1p}$ are provided in Equations (5) and (6), respectively. The entrant firm’s profit maximization problem is:

$$\max_{(p_3,u_3)} \Pi_{e,h} = N_3 \cdot p_3 - e_3 \cdot u_3;$$
\[ s.t. \quad u_3 \geq U_3 + 1/d_3. \]

It is easy to show that the optimal capacity decision satisfy \( u_3 = U_3 + 1/d_3 \).

The solution of this problem is \( p_3^* = \frac{b p_1 + (b+1) e_2 m_0 q}{2b} \) with \( p_3^* \leq p_2 \). \( \Pi_{c,b} \) is concave since \( \frac{d^2 \Pi_{c,b}}{d(p_3)^2} = \frac{2ab}{m_0(b+1)(d_2-d_1)} \) < 0. Let \( \Pi_{c,b}(p_3^*) \) be the entrant firm’s maximum profit.

The incumbent firm’s profit maximization problem is:

\[
\max_{(p_1,p_2,u_1,u_2)} \Pi_{i,b} = N_1 p_1 + N_2 p_2 - e_1 u_1 - e_2 u_2; \\
\text{s.t.} \quad u_1 \geq U_1 + 1/d_1; \\
\quad \quad \quad u_2 \geq U_2 + 1/d_2; \\
\max_{(p_3,u_3)} \Pi_{e,b} = 0.
\]

It can be easily shown that the optimal capacity levels satisfy

\[
\begin{align*}
\quad & u_1 = U_1 + 1/d_1. \\
\quad & u_2 = U_2 + 1/d_2.
\end{align*}
\]

For the Hessian matrix of \( \Pi_{c,b} \), the first principal minor is \( \frac{2abd_2}{d_1^2 m_0(b+1)(d_1-d_2)} < 0 \), and the second principal minor is \( \frac{4a^2 b^2}{d_1^2 m_0^2 (b+1)^2 (d_2-d_1)} > 0 \). Therefore, \( \Pi_{c,b} \) is concave, and the second-order condition is satisfied.

We obtain the solution of \( \Pi_{c,b}(p_3^*) = 0 \) as \( \bar{p} \), and we let \( p_2^* = \bar{p} \) to deter competitor entry. The premium-service pricing is \( p_{1b}^d = \arg \max_{p_1} \Pi_{i,b} \mid_{p_2=\bar{p}} \). Since the general solutions of equilibrium profit and pricing/capacity decisions are extremely complex, we only list the incumbent provider’s pricing decisions under \( e_1 = e_2 \) and \( v_0 = 0 \) to demonstrate the complexity of the solutions.

\[
\begin{align*}
p_{1b}^d &= \frac{(b+1)m_0[(2d_2-d_1)(e_1+k)+2d_1e_1]}{4bd_2} - \frac{d_1 a(1+b)m_0[a(1+b)m_0(e_1+k-2e_1q)^2-16be_1q]}{4abd_2}; \\
p_{2b}^d &= \frac{b+1}{8b(d_1-d_2)^2} \left\{ m_0[(k+e_1)(3d_2^2-5d_1d_2+2d_2^2)+m_0q(2d_2^2e_1-6d_1d_2e_1+4d_2^2e_1)] \right. \\
&\left. + \frac{d_1}{8b(d_2-d_1)} \sqrt{(b+1)m_0[a(1+b)m_0(e_1+k-2e_1q)^2-16be_1q]-\frac{1}{8b(d_2-d_1)}} \right\}.
\end{align*}
\]
The solution of this problem is

\[
\sqrt{\frac{2m_0(b+1)}{(d_2-d_1)}} \cdot \left\{ 8b(d_1 - 2d_2)(d_1 - d_2)e_1q - a(b+1)(d_1^3 - 3d_1^2d_2 + 4d_1d_2^2 - 2d_2^3)q_0(e_1 + k - 2e_1q)^2 
+ \sqrt{ad_1(d_1 - 2d_2)(e_1 + k - 2e_1q)(d_2 - d_1)} \sqrt{m_0(1+b)[a(1+b)m_0(e_1 + k - 2e_1q)^2 - 16be_1q]} \right\}^{1/2}. \]

Proof of Lemma 6

Using \( \mathbb{N}_3 \) and \( \mathbb{U}_3 \) from Equations (3) and (4) respectively, the entrant firm’s profit maximization problem can now be written as:

\[
\max_{(p_3, u_3)} \Pi_{e,b} = \mathbb{N}_3 \cdot p_3 - e_2 \cdot q \cdot u_3;
\]

\[s.t. \quad u_3 \geq \mathbb{U}_3 + 1/d_3.
\]

\( \Pi_{e,b} \) is concave since \( \frac{d^2 \Pi_{e,b}}{d(p_3)^2} = \frac{2ab}{m_0(b+1)(d_2-d_1)} < 0 \). The solution of this problem is \( p_3^* = \frac{b p_1 + (b+1)e_2 m_0 q}{2b} \).

Using \( \mathbb{N}_2 \) and \( \mathbb{U}_2 \) from Equations (1) and (2), respectively; and \( \mathbb{N}_{1p} \) and \( \mathbb{U}_{1p} \) from Equations (5) and (6), respectively, the incumbent firm’s profit maximization problem can be written as:

\[
\max_{(p_1, p_2, u_1, u_2)} \Pi_{i,b} = \mathbb{N}_{1p} \cdot p_1 + \mathbb{N}_2 \cdot p_2 - e_1 \cdot u_1 - e_2 \cdot u_2;
\]

\[s.t. \quad u_1 \geq \mathbb{U}_{1p} + 1/d_1;
\]

\[u_2 \geq \mathbb{U}_2 + 1/d_2.
\]

It can be easily shown that the optimal capacity levels satisfy

\[u_1 = \mathbb{U}_{1p} + 1/d_1.
\]

\[u_2 = \mathbb{U}_2 + 1/d_2.
\]

Note that \( \mathbb{N}_2 = \mathbb{U}_2 = 0 \), and that it is easy to show that the optimal capacity decision satisfy

\[u_1 = \mathbb{U}_{1p} + 1/d_1.\]

Also, note that \( \Pi_{i,b}|_{p_3=p_3^*} \) is concave in \( p_1 \) since \( \frac{d^2 \Pi_{i,b}|_{p_3=p_3^*}}{dp_1^2} = \frac{ab(2d_2-d_1)}{d_1m_0(b+1)(d_2-d_1)} < 0. \)

The solution of this problem \( p_1^* = g(p_3) \) is identical to \( p_1^* \) in Equation (EC.8). The equilibrium pricing \( p_{16}^{ad} \) and \( p_{36}^{ad} \) can be obtained by solving the two equations jointly

\[
\left\{ \begin{array}{l}
    p_1 = (1+b)d_3(e_1+k)m_0 + bd_2v_0 - d_1((1+b)km_0 + k(v_0-p_3)) \right. \\
    p_3 = \frac{b p_1 + (b+1)e_2 m_0 q}{2b}.
\end{array} \right.
\]
The solutions of the equilibrium pricing are shown in Lemma 6. The equilibrium capacity levels are:

\[ v_{1b}^{nd} = \frac{1}{bd_1(b-1)(d-1-d_2)(d_1-4d_2)} \left\{ am_0[(d_1 - 2d_2)(d_1k - d_2(e_1 + k)) - d_1d_2e_2q] 
- b(d_1 - d_2)[d_1 - 2d_2(2 + av_0)][d_1^2 + d_1d_2(-5 + am_0(e_1 - 2k + e_2q) - 2av_0) 
+ 2d_2^2(2 + a(-e_1m_0 + km_0 + v_0))] \right\}. \]

\[ v_{2b}^{nd} = 1/d_2. \]

\[ v_{3b}^{nd} = \frac{1}{bd_2(d_2 - d_1)(4d_2 - d_1)} \left\{ ad_2m_0[-d_1k + d_1e_2q + d_2(e_1 + k - 2e_2q)] 
+ b[d_1^2 - d_1d_2(5 + a(km_0 - e_2m_0q + v_0)) + d_2^2(4 + am_0(e - 1 + k - 2e_2q) + av_0)] \right\}. \]

The equilibrium profits are:

\[ \Pi_{i,b}^{nd} = -\frac{e_2}{d_2} + \frac{ad_2((1 + b)m_0(2d_2(-e_1 + k) + d_1(e_1 - 2k + e_2q)))}{b(1 + b)d_1(d_1 - 4d_2)(d_1 - d_2)m_0} \]

\[ + \frac{2b(d_1 - d_2)v_0)((1 + b)m_0(2d_2(e_1 + k)d_1(-2k + e_2q)) + 2b(-d_1 + d_2)v_0)}{b(1 + b)d_1(d_1 - 4d_2)(d_1 - d_2)m_0} \]

\[ - \frac{e_1(am_0((d_1 - 2d_2)(d_1k - d_2(e_1 + k)) - d_1d_2e_2q) - b(d_1 - d_2)(d_1 - 2d_2(2 + av_0))}{(1 + b)bd_1(d_1 - 4d_2)(d_1 - d_2)} \]

\[ + \frac{b^2(d_1^2 + d_1d_2(5 + am_0(e_1 - 2k + e_2q) - 2av_0) + d_2^2(2 + a(-e_1m_0 + km_0 + v_0))))}{(1 + b)bd_1(d_1 - 4d_2)(d_1 - d_2)}. \]

\[ \Pi_{e,b}^{nd} = -\frac{e_2q}{d_2} - \frac{a((1 + b)m_0[-d_1k + d_1e_2q + d_2(e_1 + k - 2e_2q)] + b(-d_1 + d_2)v_0)^2}{b(1 + b)(d_1 - d_2)(d_1 - 4d_2)m_0}. \]

\[ \square \]

**Proof of Proposition 4**

Note that

\[ \frac{d\Pi_{1,a}}{da} = \frac{1}{b(b^2 - 1)d_1\sqrt{ad_2e_2m_0q(d_2 - d_1)}} \left\{ -b^3(e_1 - e_2q)(-e_2m_0q + km_0 + v_0)\sqrt{ad_2e_2m_0q(d_2 - d_1)} 
+ b^2m_0(e_1 - e_2q)(e_2q - k)\sqrt{ad_2e_2m_0q(d_2 - d_1)} 
+ b\sqrt{ad_2e_2m_0q(d_2 - d_1)}(v_0(e_1 - e_2q) - e_2m_0q(e_1 - e_2q + k)) 
+ e_2m_0q(-e_1 + e_2q - k)\sqrt{ad_2e_2m_0q(d_2 - d_1)} - \sqrt{b + 1}b^{5/2}e_2q(d_1 - d_2)(m_0(e_1 - 2e_2q + k) + v_0) 
+ \sqrt{b + 1}b^{5/2}e_2qv_0(d_1 - d_2) + \sqrt{b}\sqrt{b + 1}e_2m_0q(d_1 - d_2)(e_1 - 2e_2q + k) \right\}. \]
Setting $\frac{d\Pi_{i,a}}{da} = 0$, we arrive at

$$a_c = \frac{(b - 1)^2b_{e2}q(d_2 - d_1)((b + 1)m_0(e_1 - 2e_2q + k) + bv_0)^2}{(b + 1)d_2m_0(b^2(e_1 - e_2q)(-e_2m_0q + km_0 + v_0 + bv_0(e_2q - e_1) + e_2m_0q(e_1 - e_2q + k))^2}.$$

Note that $\frac{d^2\Pi_{i,a}}{da^2} = \frac{e_2q(d_1 - d_2)[m_0(b + 1)(e_1 + k - 2e_2q) + bv_0]}{2d_1^3\sqrt{b(b + 1)[ad_2(b + 1)(d_2 - d_1)e_2m_0q]}} < 0$ when $m_0(b + 1)(e_1 + k - 2e_2q) + bv_0 > 0$. □

**Proof of Proposition 5**

(a) Note that the solutions of $\Delta\Pi_1^{d_{-nd}}(k) = 0$ are

$$k_1 = -\frac{1}{8a(b + 1)d_2^2m_0(d_1 - d_2)}\{a(b + 1)d_2e_1m_0(d_1^2(q - 1) + 4d_1d_2(1 - 3q) + 8d_2^2(2q - 1))$$

$$+ (d_1 - 4d_2)^2(d_2\Gamma + 2\sqrt{b}\sqrt{b + 1}\sqrt{ad_2e_1m_0q(d_2 - d_1)})\},$$

and

$$k_2 = \frac{1}{8a(b + 1)d_2^2m_0(d_1 - d_2)}\{(d_1 - 4d_2)^2(d_2\Gamma - 2\sqrt{b}\sqrt{b + 1}\sqrt{ad_2e_1m_0q(d_2 - d_1)})$$

$$- a(b + 1)d_2e_1m_0[d_1^2(q - 1) + 4d_1d_2(1 - 3q) + 8d_2^2(2q - 1)]\},$$

where

$$\Gamma = \sqrt{\frac{a(b + 1)d_1e_1m_0}{d_2(d_1 - 4d_2)^2}} \cdot \{a(b + 1)d_1d_2e_1m_0(q - 1)^2 - 4\sqrt{b}\sqrt{bq(d_1 - d_2)(d_1 + 8d_2)}$$

$$- \sqrt{b + 1}(q - 1)(d_1 + 4d_2)\sqrt{ad_2e_1m_0q(d_2 - d_1)}\}^{\frac{1}{2}}.$$

(b) Assuming $e_1 = e_2$ and $v_0 = 0$, we have that $\Delta\Pi_1^{d_{-nd}}$ is a second-order polynomial function of $\sqrt{m_0}$. Solving $\Delta\Pi_1^{d_{-nd}}(\sqrt{m_0}) = 0$, we arrive at two solutions that correspond to two values of $m_0$:

$$m_1 = \left(\frac{A + B}{2C}\right)^2,$$

and

$$m_2 = \left(\frac{A - B}{2C}\right)^2,$$

where

$$A = 2b^{7/2}\sqrt{b + 1}\sqrt{\frac{1}{b^4 + b^3}e_1q(d_1 - 4d_2)^2(d_1 - d_2)^2(-2e_1q + e_1 + k)},$$

and

$$B = 2b^{7/2}\sqrt{b + 1}\sqrt{\frac{1}{b^4 + b^3}e_1q(d_1 - 4d_2)^2(d_1 - d_2)^2(2e_1q + e_1 + k)}.$$
\[ B = 2\{ b^4 d_1 e_1^2 q^2 (d_1 - 4d_2)^2 (d_1 - d_2)^2 [d_1^2 (e_1 - k)^2 + d_1 d_2 (-2e_1 q + e_1 + k)(7k - e_1 (2q + 5))] \]
\[-8d_2^2 (e_1 - k)(e_1 (2q - 1) - k)]^{\frac{1}{2}}, \]

and

\[ C = (d_1 e_1 q - d_1 k + d_2 (-2e_1 q + e_1 + k))[d_1^2 e_1(q - 1) + 4d_1 d_2 (-2e_1 q + e_1 + k) - 4d_2^2 (-2e_1 q + e_1 + k)]. \]

(c) Assuming \( e_1 = e_2 \) and \( v_0 = 0 \), we have that \( \Delta \Pi_{1a}^{d - nd} \) is a second-order polynomial function of \( \sqrt{a} \). Solving \( \Delta \Pi_{1a}^{d - nd}(\sqrt{a}) = 0 \), we arrive at two solutions that correspond to two values of \( a \):

\[ a_1 = \left( \frac{A + B}{2C} \right)^2, \]

and

\[ a_2 = \left( \frac{A - B}{2C} \right)^2, \]

where

\[ A = b^2 e_1 q (d_1 - 4d_2)^2 (d_1 - d_2)^2 (-2e_1 q + e_1 + k), \]

\[ B = \{ b^4 d_1 e_1^2 q^2 (d_1 - 4d_2)^2 (d_1 - d_2)^2 [d_1^2 (e_1 - k)^2 + d_1 d_2 (-2e_1 q + e_1 + k)(7k - e_1 (2q + 5))] \]
\[-8d_2^2 (e_1 - k)(e_1 (2q - 1) - k)]^{\frac{1}{2}}, \]

and

\[ C = (d_1 (k - e_1 q) - d_2 (e_1 + k) + 2d_2 e_1 q)[d_1^2 e_1(q - 1) + 4d_1 d_2 (-2e_1 q + e_1 + k) + d_2^2 (8e_1 q - 4(e_1 + k))]. \]
Numerical study complementing Proposition 2

We start the numerical study by showing a parameter setting of the setting in Figure EC.1:

![Figure EC.1](image)

**Figure EC.1** The incumbent provider’s profit \((d_1 = 1, d_2 = 2, e_1 = 0.5, e_2 = 0.5, a = 10, m_0 = 1, v_0 = 0)\)

It is obvious that a threshold of \(b\) exists. In the remainder of the study, we continue to show that a threshold of \(b\) exists in a variety of parameter settings. First, we show three settings where the restrictions \(e_1 = e_2\) and \(v_0 = 0\) are lifted in Figures EC.2 to EC.4.

![Figure EC.2](image)

**Figure EC.2** The incumbent provider’s profit \((d_1 = 1, d_2 = 2, e_1 = 1, e_2 = 0.5, k = 3, a = 10, m_0 = 1, v_0 = 0)\)
After showing that a threshold of $b$ exists when the restrictions $e_1 = e_2$ and $v_0 = 0$ are lifted, we continue to alter other parameter values to demonstrate the robustness of the results.
When changing the value of $k$, we have Figures EC.5 and EC.6.

**Figure EC.5** The incumbent provider’s profit ($d_1 = 1$, $d_2 = 2$, $e_1 = 1$, $e_2 = 0.5$, $k = 4$, $a = 10$, $m_0 = 1$, $v_0 = 1$)

**Figure EC.6** The incumbent provider’s profit ($d_1 = 1$, $d_2 = 2$, $e_1 = 1$, $e_2 = 0.5$, $k = 2$, $a = 10$, $m_0 = 1$, $v_0 = 1$)
When changing the value of $a$, we have Figures EC.7 and EC.8.

**Figure EC.7**  The incumbent provider’s profit ($d_1 = 1$, $d_2 = 2$, $e_1 = 1$, $e_2 = 0.5$, $k = 3$, $a = 5$, $m_0 = 1$, $v_0 = 1$)

**Figure EC.8**  The incumbent provider’s profit ($d_1 = 1$, $d_2 = 2$, $e_1 = 1$, $e_2 = 0.5$, $k = 3$, $a = 20$, $m_0 = 1$, $v_0 = 1$)
When changing the value of $m_0$, we have Figures EC.9 and EC.10.

**Figure EC.9**  The incumbent provider’s profit ($d_1 = 1$, $d_2 = 2$, $e_1 = 1$, $e_2 = 0.5$, $k = 3$, $a = 10$, $m_0 = 0.5$, $v_0 = 1$)

**Figure EC.10**  The incumbent provider’s profit ($d_1 = 1$, $d_2 = 2$, $e_1 = 1$, $e_2 = 0.5$, $k = 3$, $a = 10$, $m_0 = 2$, $v_0 = 1$)

It can be concluded from observations that the existence of a threshold of $b$ is quite robust.
Note: All online references in the following list are last accessed on November 18, 2016.

References