Decision Support

Financial risk, inventory decision and process improvement for a firm with random capacity

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\textbf{ABSTRACT}

Companies strive to maximize their value on the financial market and tailor their inventory decisions to achieve this goal. In this paper, we consider a company aiming to maximize its firm value in a newsvendor setting with a randomly capacitated supplier and a stochastic demand correlated with the market return. We employ the Capital Asset Pricing Model to evaluate firm value. We demonstrate that while the optimal order quantity is independent of the supplier’s random capacity, firm value is not. Building on this firm-value/random-capacity dependence, we next explore the impact of capacity process improvements on firm value and establish when and how such improvements will contribute to firm value the most. We also identify several factors that moderate the impact of capacity process improvements on firm value. Our research results should help managers to select suppliers who would contribute to firm value maximization.

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1. Introduction

With strategies such as operational excellence (Liker & Franz, 2011), managers strive to maximize the value of their firms’ common equity on the financial market (Lazonick & O’Sullivan, 2000), commonly called market capitalization, shareholder value, or firm value. However, one of the challenges when maximizing firm value is that supply uncertainties may create mismatches between supply and demand. For example, Apple Inc. reportedly slowed down its production of the iPhone 6S Plus due to an issue regarding the production of backlight modules (CNET, 2016). Facing various uncertainties, managers can plan inventories and improve production processes in order to better match supply and demand; such a match is crucial in managing financial risks that impact firm value. For instance, over a two-day period, the mean stock market reaction is more than –6% when excess inventory occurs (Hendricks & Singhal, 2014). It is therefore vitally important for managers to identify firm-value-maximizing inventory and production process improvement decisions to best manage their risks (Aven, 2016). Such undertaking is the focus of this paper.

We consider a firm that sells an item purchased from a supply source with random available capacity, faces a newsvendor-type decision, and aims to maximize its own firm value. Most operations management models, such as the classic newsvendor model, use an expected profit criterion (Qin, Wang, Vakharia, Chen, & Seref, 2011), which may not lead to decisions that maximize firm value. Motivated by this drawback of typical operations management models, we use the Capital Asset Pricing Model (CAPM) (Lintner, 1965; Sharpe, 1964) to evaluate how operational decisions impact firm value.

The CAPM is a mathematical model that decides a theoretically appropriate rate of investment return in the financial market. The CAPM framework penalizes positive covariance of an asset’s expected return with the (broader) market return. Such positive covariance is named systematic risk or non-diversifiable risk. The intuition behind such penalty is that for investors with a well-diversified portfolio (e.g., most institutional investors), adding systematic risk amplifies the risk of their portfolio, which is not worthwhile unless the investors are sufficiently compensated by higher investment returns (or equivalently, lower current asset prices).

The use of the CAPM in operations management started with its application to the classic newsvendor model (Anvari, 1987; Kim & Chung, 1989). Later, the CAPM was applied in multi-period inventory settings such as the \((Q, r)\) policy in continuous-review inventory models (Singhal, Raturi, & Bryant, 1994) and the order-up-to-S policy in periodic-review inventory models (Jaber & Schefer, 1996). Moreover, the CAPM has been used to link a firm’s inventory parameters and policies to its opportunity cost of capital.
tied-up in inventories (e.g., Shan & Zhu, 2013; Rajagopalan, 2013). Different from the Consumption CAPM used by Berling and Rosling (2005), we do not assume identical households for the economy, which can be restrictive. For a comprehensive literature review of the operations-finance interface in general, see Zhao and Huchzermeier (2015).

Despite using the CAPM, the aforementioned studies do not incorporate randomness in the supply source, which can be significant in modern supply chains (Bollapragada, Rao, & Zhang, 2004; Chopra & Sodhi, 2004; Ray & Jenamani, 2016; Xiao & Shi, 2016). As a form of supply quantity uncertainty and often found in production systems (Ji, Wang, & Hu, 2016), capacity randomness does not depend on order quantity and may stem from random production times (Sarkar & Zangwill, 1991), machine breakdowns (Lehoczky, Sethi, Soner, & Taksar, 1991), and material shortages (Chopra & Sodhi, 2004). Carollo, Akella, and Morton (1994) consider a single-period inventory model with a single supply source which has random capacity and discover that the optimal order decision is not affected by the random capacity. Other researchers have examined settings with learning effects (Silbermayr & Minner, 2016; Vörös, 2013; Wang, Plante, & Tang, 2013), resource dependency (Kettenun & Bunn, 2016) and multiple supply sources (Dada, Petruzzi, & Schwarz, 2007; Wang, Gilland, & Tomlin, 2010; Wang, Xiao, & Yang, 2014). Although process improvement is believed to increase the firm value (Keen, 1997) and impact operational performance (Arunugam, Antony, & Linderman, 2016; Mizigier, Hora, Wagner, & Jüttner, 2015), it is unclear how or when the firm value is maximized on the financial market via process improvement. To the best of our knowledge, this paper is the first one to analyze the impact on firm value of improving a generally-distributed random capacity using the CAPM framework.

To investigate how random capacity impacts inventory decisions and firm value, we consider a scenario where the product demand is correlated with the market return. The market return is defined as the return of the portfolio that consists of all assets accessible to investors, with weights proportional to the market value of each asset. A positive correlation between demand and market return can often be found in durable-goods industries, such as the automobile industry, while a negative correlation between demand and market return exists in many low-end industries where better economic conditions mean less demand, such as basic toys targeted for low-income consumers (PConsoleMag, 2008). Similarly, if the firm and its customers are not in the same industry or in the same country, then there could potentially be a negative correlation between demand and the market return.

Our paper makes two major contributions. First, we discover that the optimal ordering quantity is generally different from that in the classic newsvendor model and does not depend on random capacity. This finding extends the results of previous studies that assume unlimited capacity (Anvari, 1987; Kim & Chung, 1989) to a more generalized setting with capacity randomness. The finding also generalizes the results of a random-capacity newsvendor with cost-minimizing objectives (Carollo et al., 1994) to situations with firm-value-maximizing objectives. In addition, although the CAPM builds on the risk-averseness of investors and demonstrates a trade-off between risk and returns, our first finding differs from prior results which say that capacity uncertainty reduces the order quantity for a risk-averse newsvendor (Wu, Zhu, & Teunter, 2013).

Establishing that firm value does depend on the supplier’s random capacity is the second major contribution of the paper. Building on this contribution, we explore the relationship between capacity process improvements and firm value. We show that capacity process improvements are generally beneficial to firm value, but a careful analysis of such benefits is needed prior to starting any capacity process improvement efforts, since diminishing returns are at play and several factors moderate such benefits. In particular, we find that some of the moderating factors are the correlation between demand and the market return, capacity scariness, and product profitability. If the random capacity is internal to the company, then our research results should help managers to determine the type and magnitude of capacity process improvement projects. If the random capacity is external to the company, then our findings should guide managers to select suppliers based on their contribution to firm value.

The paper is organized as follows. Section 2 contains the problem definition and formulation, as well as the derivation of the optimal order quantity. Section 3 investigates the role of capacity process improvements in increasing firm value. Section 4 presents analytical and numerical results regarding the moderating impact of several factors on firm value when undertaking capacity process improvement efforts. Section 5 summarizes our research results and managerial insights, and offers further research directions. To improve the flow of the paper, all the proofs are deferred to the appendix.

2. Maximizing firm value in a newsvendor setting with random capacity

2.1. A traditional newsvendor problem with random capacity

The key notations are summarized in Table 1. Consider a company in a newsvendor setting that procures and sells an item. The source of the item, from now on called the supplier, could be internal or external to the company, and has a random capacity Y with distribution F(·). Demand for the item is a random variable Z with distribution G(·). Since F(·) captures technical failures, quality problems, and exogenous factors such as the weather and catastrophic events, we assume that Y and Z are independent.

The sequence of events in procuring and selling the item is the following. First, prior to the start of the period, the company orders from the supplier Q units of the item at a unit cost of a. Second, because the supplier has random capacity, at the beginning of the period the company receives Q units if the supplier has enough capacity to produce the Q units; otherwise, the company receives whatever amount the supplier was able to produce. Third, at the beginning of the period, a payment is made for the units received. Fourth, revenue from selling the units during the period at a unit price r is received at the end of the period. Fifth, any unsold goods by the end of the period are salvaged at a discounted price s which is less than r; the revenue from salvaged units is received at the
end of the period. Sixth, any unmet demand at the end of the period is penalized at a cost of \( d \) per unit. Keep in mind that \( a \) is in beginning-of-period dollars while \( r, s, \) and \( d \) are in end-of-period dollars.

Let \( r_f \) be the risk-free interest rate. Then the end-of-period realized random profit \( D(Q) \) can be expressed as

\[
D(Q) = \left[ r - a(1 + r_f) \right] \cdot \min\{Q, Y\} - (r - s) \cdot \min\{Q, Y\} - Z^+ - d \cdot \left[ Z - \min\{Q, Y\} \right]^+ .
\]

Let \( E[{.}] \) be the expected value operator. The end-of-period expected profit \( E[D(Q)] \) is given by

\[
E[D(Q)] = \left[ r - a(1 + r_f) \right] \cdot \left\{ 1 - F(Q) \right\} Q + \int_0^Q Y f(Y) dY \\
- (r - s) \cdot \left\{ \int_0^Q (Y - Z) g(Z) dZ f(Y) dY + [1 - F(Q)] \right\} \\
\int_0^Q (Q - Z) g(Z) dZ \\
- d \cdot \left\{ \int_0^\infty (Z - Y) g(Z) dZ f(Y) dY + [1 - F(Q)] \right\} \\
\int_Q^{\infty} (Q - Z) g(Z) dZ .
\]

Following the traditional approach of optimizing an expected value, it is easy to derive from Eqs. (2) and (A.3) in the appendix that the order quantity maximizing \( E[D(Q)] \), denoted by \( Q_c \), is given by

\[
Q_c = \arg \max_Q E[D(Q)] = \max_{Q} \left\{ 1 - (r - s) \right\} \left( r - a(1 + r_f) + d \right) .
\]

The expression of \( Q_c \) in Eq. (3), while similar to Eq. (3) in Khoura (1999), extends Proposition 1 in Cariallo et al. (1994) from minimizing expected holding and back-ordering cost with no time discounting to maximizing expected profit with time discounting. Of special interest is that in our formulation, \( Q_c \) is independent of the random capacity, as was found in Cariallo et al. (1994).

### 2.2. Maximizing firm value under the CAPM framework

We now consider the more general situation where the company in Section 2.1 has a random demand which is correlated with the market return and the company aims to maximize its firm value. Anvari (1987) and Kim and Chung (1989) demonstrated that under the CAPM and its additivity property (Thorstenson, 1988), maximizing the firm value by ordering \( Q \) units is equivalent to maximizing the market value of the end-of-period realized random cash flow obtained by ordering \( Q \) units, which in our model is represented by \( D(Q) \).

To determine the market value of \( D(Q) \), we need to simplify the expression of \( D(Q) \) in Eq. (1). Let \( U(Q) = \min\{Y, Q\} \) denote the quantity of units received at the beginning of the period after ordering \( Q \) units. Then the beginning-of-period negative cash flow is \(-aU(Q)\), and the equivalent end-of-period negative cash flow is \( D_1(Q) = -a(1 + r_f)U(Q) \). At the same time, it is not hard to show that the end-of-period random revenue \( V(Q) \) is given by

\[
V(Q) = \begin{cases} \{ru(Q) - d[Z - U(Q)] \}, & \text{if } Z \geq U(Q) ; \\
ru(Q) - (r - s)[U(Q) - Z], & \text{if } Z < U(Q). \\
\end{cases}
\]

Consequently, it is easy to see that \( D(Q) \) in Eq. (1) can be concisely restated as \( D(Q) = D_1(Q) + V(Q) \). Now, according to the CAPM, the market value of \( D(Q) \) is equal to the expected value of \( D(Q) \) minus the penalty for the systematic risk of \( D(Q) \), which is defined as \( \Omega \text{Cov}(D(Q), M) \), when \( \text{Cov}(\cdot, \cdot) \) is the covariance operator, \( \Omega \) is the market price per unit of risk, and \( M \) is the market return (see Table 1). The beginning-of-period equivalent of the market value of \( D(Q) \), which is denoted by \( S(Q) \), then becomes

\[
S(Q) = (1 + r_f)^{-1}[E(D(Q)) - \Omega \text{Cov}(D(Q), M)] .
\]

From now on, \( S(Q) \) will be used as a surrogate for firm value.

### 2.3. Finding the order decision that maximizes firm value

To gain analytical insight, we will assume that the demand \( Z \) and the market return \( M \) are jointly normally distributed, with respective parameters \((\mu_Z, \sigma_Z^2) \) and \((\mu_M, \sigma_M^2) \). Let \( \phi(\cdot) \) and \( \Phi(\cdot) \) respectively denote the probability density function and the cumulative distribution function of the standard normal distribution.

Since as alluded earlier the capacity randomness is due to technical failures and other exogenous factors, \( Y \) and \( M \) are independent, namely \( \text{Cov}(Y, M) = 0 \). We assume that \( f(Y) \) is continuous and has support \((\min, \max) \), but it is otherwise generally distributed. Let \( \delta_{MZ} \) be the market-demand correlation, where \( \delta_{MZ} \) is the correlation coefficient between random variables \( A \) and \( B \), and let \( s_k \) be the Sharpe ratio (see Table 1). The firm-value-maximizing order decision \( Q^* \), from now on called the optimal order quantity, is characterized in Lemma 1.

**Lemma 1.** \( Q^* = \mu_Z + q^*_2 \sigma_Z \), where \( q^*_2 \) satisfies the equation

\[
\Phi(q^*_2) + s_k \delta_{MZ} \phi(q^*_2) = \frac{r - a(1 + r_f) + d}{r - s + d} .
\]

**Lemma 1** provides a necessary condition for the optimal order quantity \( Q^* \). In **Lemma 2**, we present an important result regarding \( Q^* \) and a second-order condition (SOC) for the optimality of \( Q^* \).

**Lemma 2.** When \( Q^* < Y_{max} \), \( Q^* \) satisfies the SOC \( \frac{\partial^2 S(Q^*)}{\partial Q^2} < 0 \).

Building on **Lemmata 1 and 2**, we are now prepared to establish in **Theorem 1** the conditions under which \( Q^* \) is unique or trivially non-unique.

**Theorem 1.** \( Q^* \) characterized by Eq. (5) is the optimal order quantity.

(a) When \( Q^* < Y_{max} \), \( Q^* \) is the unique optimal order quantity.

(b) When \( Q^* \geq Y_{max} \), \( Q^* \) is optimal, but it is trivially non-unique because any \( Q \geq Y_{max} \) would also be optimal.

From **Theorem 1**, we know that when \( Q^* \geq Y_{max} \), the optimal decision is trivial, since placing an order for \( Y_{max} \) units will suffice. For that reason, in the reminder of the paper, we will focus on the non-trivial situations when \( Q^* < Y_{max} \) and hence the optimal decision is to order \( Q^* \) units.

We would like to first point out that when comparing the work in Anvari (1987) and in Kim and Chung (1989) to our work, they consider neither a penalty per unmet demand \( d \) nor a supplier with finite random capacity and support \((\min, \max) \). However, their major result is identical to **Lemma 1** after substituting \( d = 0 \) in Eq. (5). On one hand, the coincidence confirms that the firm-value-maximizing order decision not only depends on the same economic parameters already present in the classic newsvendor model (and in our case on the additional economic parameter \( d \)), but also depends on the market-demand correlation as well as on a systematic risk penalty captured by Sharpe’s ratio \( s_k \). On the other hand, the coincidence surprisingly implies that the supply source plays no role in the determination of the optimal order quantity. In other words, the decision maker should order \( Q^* \) units regardless of whether the supply source is unlimited or the supply source is capacitated and random.

We would next like to point out that when comparing our work to the work in Cariallo et al. (1994), their demand is not correlated to market return, and they minimize the expected holding and
back-ordering cost (which is essentially equivalent to maximizing expected profit). However, one of Carollo et al.’s major results is identical to Lemma 1 after substituting $d = 0$, $r_f = 0$, and $\delta_{MZ} = 0$. The similarity implies that the firm-value-maximizing order quantity will generally be higher/lower than the expected-profit-maximizing order quantity, depending on the market-demand correlation being positive/negative.

On a related matter, Wu et al. (2013) consider a risk-averse newsvendor with random capacity and find that the optimal order quantity is not independent of random capacity; specifically, they find that capacity uncertainty decreases the optimal order quantity. We find it intriguing that for the cost-minimizing newsvendor with random capacity in Carollo et al. (1994) and the firm-value-maximizing newsvendor with random capacity we consider, the optimal order quantity is independent of random capacity, but for the risk-averse newsvendor with random capacity in Wu et al. (2013), such independence does not exist.

At the same time, when compared to Wu et al. (2013), we find our result intriguing because the CAPM framework builds on the risk-averseness of investors and demonstrates a trade-off between risk and return. Nevertheless, in our case the risk-averseness assumed in the CAPM framework did not lead to an impact of capacity uncertainty on the optimal order quantity, which obviously is opposite to the result in Wu et al. (2013).

Due to the classic newsvendor model’s omnipresence and historical importance, we now want to compare our $Q^*$ to the optimal order quantity in the classic newsvendor model. To facilitate the comparison, we will add to the classic newsvendor model consideration of a penalty per unmet demand $d$ and an interest cost $r_f$. It is not hard to show that after adding the extra terms the critical fractile equation becomes $c_f = \frac{\mu_f}{f(\frac{\mu_f}{\sigma_f})}$, and that the optimal order quantity is identical to $Q_c$ in Eq. (3). It is now easy to see from Eq. (5) that the firm-value-maximizing order quantity will generally be higher/lower than the expected-profit-maximizing order quantity, depending on the market-demand correlation being positive/negative. This observation along with related results regarding $Q^*$ and $Q_c$ are formalized in Corollary 1.

**Corollary 1.** The following results are derived from Theorem 1 and the classic newsvendor model.

(a) $Q^*$ moves away from $Q_c$ as $S_k$ increases.

(b) $Q^*$ moves away from $Q_c$ as $[\delta_{MZ}]$ increases.

(c) As $\sigma_f$ increases, both $Q^*$ and $Q_c$ move away from $\mu_f$, and $|Q^* - Q_c|$ increases.

(d) As $c_f$ increases, both $Q^*$ and $Q_c$ increase.

**Corollary 1** (a) indicates that as the systematic risk penalty measured by $S_k$ increases, $Q^*$ in our model will change accordingly to optimize the firm’s risk profile and will do so by increasingly departing from $Q_c$. In turn, **Corollary 1(b)** suggests that because a larger association between demand and market return $[\delta_{MZ}]$ makes the firm more susceptible to systematic risk emanated from the financial market, $Q^*$ in our model will change accordingly to optimize the firm’s risk profile and will do so by increasingly departing from $Q_c$. In short, when the impact of systematic risk rises due to increases in $S_k$ and $[\delta_{MZ}]$, the optimal order quantity that managers should select will increasingly be different from $Q_c$.

If now demand risk increases due to an increase in $\sigma_f$, then **Corollary 1(c)** shows that the inventory buffers or gaps measured by $|Q^* - \mu_f|$ and $|Q_c - \mu_f|$ will increase as well. But interestingly enough, despite both $Q^*$ and $Q_c$ moving in the same direction, **Corollary 1(c)** also shows that $Q^*$ will increase faster than $Q_c$ as $\sigma_f$ increases. This suggests that as demand risk increases, managers should compensate by increasing the optimal order quantity. However, the optimal order quantity increase would be higher when financial risk is considered than when financial risk is not considered. **Corollary 1(d)** lastly indicates that as the critical fractile $c_f$ increases due to higher profitability, then both optimal order quantities $Q^*$ and $Q_c$ will increase in an effort to provide a higher service level. In other words, profitability improvements have a similar effect on both optimal order quantities.

We want to point out that our results in (b), (c), and (d) of **Corollary 1** generalize the similar results in Kim and Chung (1989) to our more general formulation which includes a supplier with random capacity, a penalty for unmet demand, and the possibility of a negative market-demand correlation.

### 3. Capacity process improvement and firm value increase

In **Section 2**, we established the optimal order quantity $Q^*$ and its corresponding maximum firm value as measured by $S(Q^*)$. A **firm value increase** will be defined as an increase in $S(Q^*)$ due to capacity process improvements. In this section, we study the relationship between capacity process improvements and firm value increases. We will provide later a formal definition of capacity process improvement, but intuitively speaking, capacity process improvement means a “better” probability distribution of capacity is attained, such as an increase in the value of $Y_{min}$. Consequently, our analysis will only consider those instances when capacity process improvement matters; i.e., when $Q^* > Y_{min}$.

Let $P(Y)$ denote the marginal contribution to firm value for each additional unit of capacity. It can be shown that

$$
P(Y) = (1 + r_f)^{-1}[r - \alpha(1 + r_f)]Y - (r - s) \int_0^Y (Y - Z)g(Z)dZ - d \int_0^{\infty} (Z - Y)g(Z)dZ - (r - s + d)\delta_{MZ}S_k\Phi\left(\frac{Y - \mu_f}{\sigma_f}\right).$$

It is not hard to demonstrate that Eq. (4) can be rewritten as

$$S(Q) = P(Q)[1 - F(Q)] + \int_0^Q P(Y)f(Y)dY.$$  

In our analysis of the role of capacity process improvement on firm value, we will look at two types of capacity process improvement. First, we will consider capacity process improvements in which the mean capacity increases but the shape of the p.d.f. of capacity does not change. This type of capacity process improvement will be called mean-capacity improvement.

Second, we will consider capacity process improvements in which the mean capacity stays the same but the variability of the capacity decreases. This type of capacity process improvement will be called capacity-variance reduction. To model capacity-variance reduction, we will use the transformation $f(Y) = b[I\mu_f + b(Y - \mu_f)]$, where $b > 0$. In this transformation, when $b > 1$ the p.d.f. of the random capacity shrinks toward its mean, when $b = 1$ the p.d.f. of the random capacity remains unchanged, and when $0 < b < 1$ the p.d.f. of the random capacity expands away from its mean. In this paper, we only consider cases where $b > 1$, i.e., the capacity variance is reduced.

In **Proposition 1**, we analyze how mean-capacity improvements and capacity-variance reductions contribute to firm value increases.

**Proposition 1.** Regarding how capacity process improvement impacts firm value increase:

(a) Mean-capacity improvements lead to firm value increases.

(b) When $\delta_{MZ} \in [0, 1]$,

(i) There exist diminishing returns to mean-capacity improvements, and firm value increases in (a) are upper-bounded by the firm value under unlimited capacity.

(ii) Capacity-variance reductions lead to firm value increases.
(iii) There exist diminishing returns to capacity-variance reductions, and firm value increases in (ii) are upperbounded by the firm value under deterministic capacity.

Because in many real-world situations random capacity is modeled by a normal distribution, the implication of Proposition 1 for these situations is that earlier capacity process improvements will contribute more to firm value increases than later ones, and eventually these contributions will become negligible as the firm value under unlimited capacity is approached.

It is worth noting that the condition in Proposition 1(c) is independent of the economic parameters included in $c_r$, meaning that it is only required that the firm’s risk profile captured by the parameters $\mu_z$, $\sigma_z$, $\delta_{MZ}$, and $s_R$ satisfies such sufficient condition. In particular, as $\sigma_z$ increases, $\delta_{MZ}$ increases, $\mu_z$ decreases, and $s_R$ decreases, it becomes easier to satisfy the sufficient conditions in Proposition 1(c). Interestingly enough, in our numerical study reported in Section 4, we find several instances when $\delta_{MZ}$ equals or is close to −1, the results of Proposition 1 hold true despite the sufficient condition not being satisfied.

4. Factors impacting the benefits of capacity process improvement on firm value

In Section 3, we investigated firm value increases resulting from capacity process improvement. In this section, we will study several factors that moderate such firm value increases. The influence of these factors will be captured in several analytical results and illustrated via numerical studies. We define capacity scarce ness as the ratio of average demand to average capacity, and we will denote it by $\rho$; i.e., $\rho = \mu_z/\mu_V$. To account in our numerical studies for different scenarios of supply, demand, capacity scarce ness and product profitability, we considered combinations of the following parameter values: $\mu_z = 10,000; \sigma_z = 1,000, 2,000$ or $3,000; \mu_V = 100,000$ or $10,000; \sigma_V/\mu_V=0.1, 0.2, 0.3; \rho = 0.1$ or 1.0; $r = 100$; $a = 10, 50$ or 90; $s = 0, 40$ or 80; $d = 0$; and for obvious reasons, we only considered combinations in which $s < a$. Although our prior analytical results apply to a generally distributed random capacity, to perform the numerical studies, we assumed a normal distribution. Lastly, as a proxy for the market return, we used the return of large-company stocks (e.g. the S&P 500 Index), and as a proxy for the risk-free interest rate, we used the return of 10-year U.S. Treasury bonds. Specifically, we obtained from Morningstar Inc. (2012, p.53) that $r_M = 11.8\%$, $\sigma_M = 20.3\%$, and $r_f = 3.6\%$. Some results of our numerical studies are presented in Figs. 1–3 which will be interpreted and used later in this section.

We focus on the following questions in our analysis:

1. What is the influence of financial risk, measured by the market-demand correlation ($\delta_{MZ}$) and Sharpe’s ratio ($s_R$), on firm value increases?
2. What is the influence of the capacity scarce ness, measured by ($\rho$), on firm value increases?
3. What is the influence of the economic factors, measured by the critical fractile ($c_r$), on firm value increases?

4.1. Financial risk and firm value increases

In our problem setting, financial risk is modeled as the interaction of the parameters $\delta_{MZ}$ and $s_R$. The moderating effects of financial risk on firm value increases are captured in Corollary 2.

Corollary 2. Financial risk moderates firm value increases as follows.

(a) Regarding mean-capacity improvements:

(i) Higher market-demand correlations ($\delta_{MZ}$) dampen firm value increases.

(ii) Higher Sharpe’s ratios ($s_R$) amplify firm value increases under negative market-demand correlations and dampen firm value increases under positive market-demand correlations.

(b) Regarding capacity-variance reductions:

(i) If $Q^* \leq \mu_V$, then higher market-demand correlations ($\delta_{MZ}$) dampen firm value increases.

(ii) If $Q^* \leq \mu_V$, then higher Sharpe’s ratios ($s_R$) amplify firm value increases under negative market-demand correlations and dampen firm value increases under positive market-demand correlations.
Since Sharpe’s ratio measures the average return earned in excess of the risk-free interest rate per unit of volatility, Corollary 2 suggests that managers would prefer to pursue capacity process improvements for items with smaller market-demand correlations and when the market provides high average returns for taking risks. The moderating effect of $\delta_{MZ}$ on firm value increase can be observed in Fig. 1. For example, the result of capacity process improvement would be a move from $\sigma_Y/\mu_Y = 0.3$ to $\sigma_Y/\mu_Y = 0.2$ (via mean-capacity improvements or capacity-variance reductions). Once can see that as predicted in Corollary 2, the increases in $S(Q^*)$ when moving from $\sigma_Y/\mu_Y = 0.3$ to $\sigma_Y/\mu_Y = 0.2$ get smaller when $\delta_{MZ}$ goes from $-1$ to $0$ to $1$.

4.2. Capacity scarcity and firm value increases

Intuitively speaking, the financial market should reward more, via firm value increases, the capacity process improvements in companies with scarce capacity than in companies with plenty of capacity. This moderating effect of capacity scarcity ($\rho$) on firm value increases is formalized in Corollary 3.

**Corollary 3.** Capacity scarcity moderates firm value increases as follows.

(a) When $\delta_{MZ} \in [0, 1]$, capacity scarcity ($\rho$) amplifies firm value increases for both mean-capacity improvements and capacity-variance reductions.

(b) When $\delta_{MZ} \in [-1, 0)$, under the sufficient condition $Y_{\min} > \mu_Z + \frac{\sigma_Y}{\delta_{MZ}^2}$, capacity scarcity ($\rho$) amplifies firm value increases for both mean-capacity improvements and capacity-variance reductions.

The moderating effect of ($\rho$) on firm value increases can be observed in Fig. 2. For example, one can see that as predicted in Corollary 3, the increases in $S(Q^*)$ when moving from $\sigma_Y/\mu_Y = 0.3$ to $\sigma_Y/\mu_Y = 0.2$ are bigger when $\rho = 1$ than when $\rho = 0.1$, regardless of the value of $\delta_{MZ}$.

4.3. The critical fractile and firm value increases

Note that as the critical fractile gets higher, the economic consequences of not receiving an ordered unit also gets higher. The moderating effects of the critical fractile ($C_F$) on firm value increase are captured in Corollary 4.

**Corollary 4.** A higher critical fractile amplifies firm value increases.

Since capacity process improvements will increase the probability that an ordered unit will be received, as the economic importance of those units gets higher, then the firm value increases will be amplified. It is easy to calculate from $C_F = \frac{q - a(1 + r_f) \sigma_d}{\gamma + d}$ that the item in Fig. 2 has a $C_F = 0.338$ and the item in Fig. 3 has a $C_F = 0.897$. The moderating effect of $C_F$ on firm value increases can be observed by comparing Figs. 2 and 3 (note that the scale of the vertical axis in Fig. 3 is different from that in Fig. 2). For example, as predicted by Corollary 3, the increases in $S(Q^*)$ when moving from $\sigma_Y/\mu_Y = 0.3$ to $\sigma_Y/\mu_Y = 0.2$ are higher in Fig. 3 than those in Fig. 2 for all six corresponding curves in both figures.

We will close this section by commenting that Corollaries 3 and 4 coincide with the following empirical findings. On one hand, small and medium-sized firms, which typically have lower margins and lower capacity cost, focus on reactive measures such as overcapacity and safety stock in production to cope with supply chain risk. On the other hand, large firms, which typically have higher margins and higher capacity cost, tend to focus on preventive measures such as strategic supplier development including capacity process improvements (Thun, Drüke, & Hoening, 2011).

5. Summary and conclusions

We considered a company aiming to maximize its firm value in a newsvendor setting with a randomly capacitated supplier and a stochastic demand correlated with the market return. The CAPM framework was used to model the firm value. We derived the optimal order quantity and established that, when trivial situations are excluded, the optimal order quantity is unique. To our surprise, we found that the optimal order quantity is independent of the random capacity. Such independence is particularly important when the supplier is external to the company and thus information on the random capacity is difficult to obtain.

To provide insight to managers, we demonstrated the impact on the optimal order quantity of variations in the values of several parameters in our formulation. Because of the classic newsvendor’s omnipresence and historical importance, using the same variations we compared our optimal order quantity to its counterpart for the classic newsvendor model. The comparisons suggested that in general, intuition gained from the classic newsvendor model will not work in our problem setting.

We next looked at the relationship between capacity process improvements and firm value. Although capacity randomness does not affect the optimal order quantity, our second major research result is that mean-capacity improvements and capacity-variance reductions do increase firm value. Moreover, we established that these two types of capacity process improvements exhibit diminishing returns and hence initial improvement efforts are more valuable than later ones. Lastly, we studied several factors that moderate the impact of capacity process improvements on firm value. The main takeaway is that sometimes the moderating effects are intuitive and sometimes they are not.

Our results should be useful to managers when prioritizing investments in capacity process improvements in order to increase firm value. When the supplier is external to the company, our findings could guide managers when selecting suppliers. On the other side, our findings should also help suppliers to engage in value-based differentiation such as capacity process improvements (Ulaga & Eggert, 2006).

Future research may proceed along several paths. One would be to use the CAPM framework in other supply chain settings. Another one would be to study rationalization of supplier improvement efforts under a firm value maximization objective. Supplier selection not only based on unit cost but also on capacity randomness would be a third future research path.

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**Appendix A.**

**Proof of Lemma 1**

**Proof.** The first-order derivative of Eq. (4) is given by

$$(1 + r_f) \frac{d}{dQ} S(Q) = [1 - F(Q)](r - a(1 + r_f) + d - (r - s + d)\Phi(q_Z) - (r - s + d)\mu\operatorname{Cov}(Z, M)\Phi(q_Z)/\sigma_Z).$$

(A.1)

In the remainder of this proof, we will derive Eq. (A.1).

Recall that $Y \sim F_Y$, where $F_Y$ is a general distribution with $\mu_Y = 0$. Also recall that $Z \sim G_Z$, where $\mu(Z, M) \neq 0$. Let $R(Q) = U(Q) - Q = \min\{Y - Q, 0\}$.

We know from Anvari (1987) that
\[
\text{Cov}(U(Q), M) = \text{Cov}(R(Q) + Q, M) = \text{Cov}(R(Q), M)
\]
\[= F(Q) \cdot \text{Cov}(Y, M) = 0.\]

From \(D_t(Q) = -a(1 + r_f)U(Q)\), it follows that
\[
\text{Cov}(D_t(Q), M) = -a(1 + r_f)\text{Cov}(U(Q), M) = 0.
\]

To maximize \(S(Q)\), we need to find the expression of \(\frac{d \text{Cov}(W(Q), M)}{dQ}\). We will begin by obtaining \(\text{Cov}(V(Q), M)\). Define an auxiliary function \(W(Q)\) where
\[
W(Q) = \begin{cases} 
-d(Z - Q), & \text{when } Y \geq Q \text{ and } Z \geq Q; \\
-d(Z - Y), & \text{when } Y < Q \text{ and } Z \geq Y; \\
(r - s)(Z - Q), & \text{when } Y \geq Q \text{ and } Z < Q; \\
(r - s)(Z - Y), & \text{when } Y < Q \text{ and } Z < Y.
\end{cases}
\]

Observe that \(V(Q) = rU(Q) + W(Q)\), we have
\[
\text{Cov}(V(Q), M) = r \cdot \text{Cov}(U(Q), M) + \text{Cov}(W(Q), M) = \text{Cov}(W(Q), M).
\]

In turn,
\[
\text{Cov}(W(Q), M) = E\left[\left(W(Q) - E(W(Q))\right)\left[M - E(M)\right]\right]
\]
\[= E\left[W(Q)\right]M - E(M)\right] - E(W(Q))E[M - E(M)]
\]
\[= E\left[W(Q)\right]M - E(M)], \quad (A.2)
\]

where
\[
E[W(Q)|M - E(M)] = E[W(Q)|M - E(M)]|_{Y \geq Q, Z \geq Q}
\]
\[+ E[W(Q)|M - E(M)]|_{Y < Q, Z} + E[W(Q)|M - E(M)]|_{Y \geq Q, Z < Q}
\]
\[+ E[W(Q)|M - E(M)]|_{Y < Q, Z < Y},
\]

and each summand corresponds to one scenario of \(W(Q)\). Denote \(y_d = [Y - \mu_y]/\sigma_y\) and \(z_d = [Q - \mu_z]/\sigma_z\). Let \(m, z, y\) be the normalized standard normal variables of \(M, Z, Y\), respectively. We will now consider each of the four scenarios of \(W(Q)\) as follows.

1) When \(Y \geq Q\) and \(Z \geq Q\), we have
\[
E\left[W(Q)\right]M - E(M)]|_{Y \geq Q, Z \geq Q} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (Z - Q) \cdot \sigma_m \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]
\[= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (Z - Q) \cdot \sigma_m \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]
\[= \sigma_m \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (Z - Q) \cdot \delta_{mz} \cdot f(Y) \cdot dY \cdot dZ
\]
\[= \delta_{mz} \cdot \sigma_m \cdot \int_{-\infty}^{+\infty} (Z - Q) \cdot f(Y) \cdot dY \cdot dZ
\]
\[= \delta_{mz} \cdot \sigma_m \cdot \int_{-\infty}^{+\infty} (Z - Q) \cdot f(Y) \cdot dY \cdot dZ
\]
\[= \delta_{mz} \cdot \sigma_m \cdot \int_{-\infty}^{+\infty} (Z - Q) \cdot f(Y) \cdot dY \cdot dZ
\]
\[= \delta_{mz} \cdot \sigma_m \cdot \int_{-\infty}^{+\infty} (Z - Q) \cdot f(Y) \cdot dY \cdot dZ
\]

2) When \(Y < Q\) and \(Z \geq Q\), we have
\[
E\left[W(Q)\right]M - E(M)]|_{Y < Q, Z \geq Q} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (Z - Q) \cdot \sigma_m \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]
\[= \sigma_m \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (Z - Q) \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]
\[= \sigma_m \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (Z - Q) \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]
\[= \sigma_m \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (Z - Q) \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]
\[= \sigma_m \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (Z - Q) \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]

Revisiting Eq. (A.2), we get
\[
\text{Cov}(W(Q), M) = \text{Cov}(Z, M) - d[1 - F(Q)] \cdot [1 - \Phi(q_d)]
\]
\[= d \int_{0}^{Q} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} (Z - Y) \cdot \sigma_m \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]
\[= \sigma_m \int_{0}^{Q} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} (Z - Y) \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]
\[= \sigma_m \int_{0}^{Q} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} (Z - Y) \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]
\[= \sigma_m \int_{0}^{Q} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} (Z - Y) \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]
\[= \sigma_m \int_{0}^{Q} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} (Z - Y) \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]
\[= \sigma_m \int_{0}^{Q} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} (Z - Y) \cdot f(m, Y, Z) \cdot dY \cdot dZ
\]

Having obtained the expression of \(\text{Cov}(W(Q), M)\), we will now derive the expression of \(\frac{d \text{Cov}(W(Q), M)}{dQ}\). For this purpose, we need the expression of \(\frac{d \text{Cov}(W(Q), M)}{dQ}\), which is given by
\[
\frac{d \text{Cov}(W(Q), M)}{dQ} = \text{Cov}(Z, M) \cdot \left\{ \frac{d[f(Q)]}{dQ} - \frac{d[\Phi(q_d)]}{dQ} \right\}
\]
\[+ \frac{d[1 - F(Q)]}{dQ} \cdot \frac{d[1 - \Phi(q_d)]}{dQ}
\]
\[+ \frac{(r - s)}{dQ} \left\{ \frac{f(Q)}{\Phi(q_d)} - \frac{f(Q) - [1 - F(Q)]}{\Phi(q_d)} \right\}
\]
\[+ \frac{(r - s)}{dQ} \left\{ \frac{f(Q)}{\Phi(q_d)} - \frac{f(Q) - [1 - F(Q)]}{\Phi(q_d)} \right\}
\]
\[+ \frac{(r - s)}{dQ} \left\{ \frac{f(Q)}{\Phi(q_d)} - \frac{f(Q) - [1 - F(Q)]}{\Phi(q_d)} \right\}
\]
\[+ \frac{(r - s)}{dQ} \left\{ \frac{f(Q)}{\Phi(q_d)} - \frac{f(Q) - [1 - F(Q)]}{\Phi(q_d)} \right\}
\]
\[+ \frac{(r - s)}{dQ} \left\{ \frac{f(Q)}{\Phi(q_d)} - \frac{f(Q) - [1 - F(Q)]}{\Phi(q_d)} \right\}
\]
\[+ \frac{(r - s)}{dQ} \left\{ \frac{f(Q)}{\Phi(q_d)} - \frac{f(Q) - [1 - F(Q)]}{\Phi(q_d)} \right\}
\]
\[d \frac{\partial}{\partial Q} E(W(Q)) = d \frac{\partial}{\partial Q} \int_0^\infty (Z - Y)g(Z)dYf(Y) \]

The proof of Eq. (A.1) is now completed. Note that \( \frac{d}{\partial Q} S(Q^*) = 0 \) when \( Q^* \geq Y_{\text{max}} \) and \( \bar{Q}_2 = [Q^* - \mu_Z]/\sigma_Z \). Lemma 1 follows from the first-order condition (FOC) \( \frac{d}{\partial Q} S(Q^*) = 0 \) and the fact that \( 1 - F(Q^*) > 0 \). \( \square \)

Proof of Lemma 2

Proof. Knowing from Lemma 1 that \( Q^* \) satisfies Eq. (5), then the second-order derivative of \( S(Q) \) at \( Q = Q^* \) becomes

\[
\frac{d^2}{dQ^2} S(Q^*) = -f(Q^*) \left( r - a(1 + r_f) + d - (s - d) \right) \cdot \frac{Q^* - \mu_Z}{\sigma_Z^2} - (s - d) \mu_Y \frac{\phi(\mu_Y/\sigma)}{\phi(0)} - \frac{Q^* - \mu_Z}{\sigma_Z^2} \cdot \frac{1}{\Omega_2} \cdot \frac{d}{\partial Q} S(Q^*). \tag{A.4}
\]

Observing that \( Q^* \) is identical to \( Q^* \) in Chung (1990), we know from Chung (1990) that

\[ 1 + \Omega \text{Cov}(Z, M) \frac{\mu_Y - Q^*}{\sigma_Y^2} > 0. \]

Since \( 1 - F(Q^*) > 0 \), then \( \frac{d^2}{dQ^2} S(Q^*) < 0 \). Hence the second-order condition is satisfied. \( \square \)

Proof of Theorem 1

Proof. Suppose there exists two stationary points, denoted as \( Q_t \) and \( Q_0 \), which satisfy Lemma 1 and Lemma 2. In addition, suppose there is no other stationary point between \( Q_t \) and \( Q_0 \) that satisfies Lemma 1 and Lemma 2. Without loss of generality, let \( Q_t < Q_0 \). From the FOC and the SOC we have that there must exist \( \epsilon_1 \), \( \epsilon_2 > 0 \) such that \( \frac{d}{\partial Q} S(Q_0 + \epsilon_1) < 0 \) and \( \frac{d}{\partial Q} S(Q_0 - \epsilon_2) > 0 \), where \( \epsilon_1, \epsilon_2 \) are small. It follows from the continuity of \( \frac{d}{\partial Q} S(Q) \) that there must exist a point \( Q_0 \) where \( Q_0 + \epsilon < Q_t < Q_0 - \epsilon \) and \( \frac{d}{\partial Q} S(Q) = 0 \). Now \( Q_0 \) satisfies the SOC in Lemma 1 and the SOC in Lemma 2. However, \( Q_0 \) satisfying the SOC contradicts the assumption that there is no other stationary point between \( Q_t \) and \( Q_0 \) satisfying both the FOC and the SOC. Therefore, \( Q^* \) has to be unique.

Now note that \( \frac{d}{\partial Q} S(Q) |_{Q=0} > 0 \) and \( \lim_{Q \to -\infty} \frac{d}{\partial Q} S(Q) < 0 \). As \( S(Q) \) is a single-variable function, when \( Q^* < Y_{\text{max}} \) we conclude that \( Q^* \) is the unique optimal order quantity and \( S(Q^*) \) is the global maximum. Lastly, when \( Q^* \geq Y_{\text{max}} \), \( Q^* \) is still optimal but trivially non-unique because any \( Q \geq Y_{\text{max}} \) will be optimal. \( \square \)

Proof of Corollary 1

Proof. This proof is based on the implicit function theorem (Chiang, 1984, p. 208). Denote

\[ \Pi = \Phi(q_2^*) + s_k \delta_{MZ} \phi(q_2^*) - c_t = 0. \]

We have that

\[ \frac{d\Pi}{dQ^*} = -\left( r - s + d \right) f(Q^*) \frac{d^2}{dQ^2} S(Q^*) > 0 \]

from Eq. (A.4) and Lemma 2.
(a) Since \( \partial \Pi / \partial s_k = \delta_{MZ} \phi(q^*_k) \), we obtain that
\[
\frac{dQ^*}{ds_r} = -\frac{\partial \Pi / \partial s_r}{\partial \Pi / \partial Q^*} = -\frac{\delta_{MZ} \phi(q^*_k)}{-\partial \Pi / \partial Q^*}.
\]
We then have that if \( Q^* < Q_c \) and \( \frac{dQ^*}{ds_r} < 0 \) when \( \delta_{MZ} > 0 \), and that if \( Q^* > Q_c \) and \( \frac{dQ^*}{ds_r} > 0 \) when \( \delta_{MZ} < 0 \), meaning that \( Q^* \) moves further away from \( Q_c \) as \( s_r \) increases in either case.

(b) Since \( \partial \Pi / \partial s_k = s_k \phi(q^*_k) \), we obtain that
\[
\frac{dQ^*}{ds_mz} = -\frac{\partial \Pi / \partial s_mz}{\partial \Pi / \partial Q^*} = s_k \phi(q^*_k) \frac{s_k}{-\partial \Pi / \partial Q^*} < 0.
\]
It follows that when \( \delta_{MZ} > 0 \), \( Q^* > Q_c \) and \( \frac{dQ^*}{ds_mz} < 0 \), and that when \( \delta_{MZ} < 0 \), \( Q^* < Q_c \) and \( \frac{dQ^*}{ds_mz} > 0 \), meaning that \( Q^* \) moves further away from \( Q_c \) as \( |\delta_{MZ}| \) increases in either case.

(c) Since
\[
\frac{\partial \Pi}{\partial s_q} = \phi(q^*_k)(-q^*_k/\sigma_q) - s_k \delta_{MZ} q^*_k \phi(q^*_k)(-q^*_k/\sigma_q)
\]
and
\[
\frac{\partial \Pi}{\partial Q^*} = \phi(q^*_k)/\sigma_q [1 + \delta_{MZ} (Z, M, Z)^* Q^*/\sigma_q],
\]
we obtain that
\[
\frac{dQ^*}{ds_q} = \frac{\partial \Pi / \partial s_q}{\partial \Pi / \partial Q^*} = q^*_k = (Q^* - \mu_q)/\sigma_q.
\]
Similarly,
\[
\frac{dQ^*}{ds_q} = (Q^* - \mu_q)/\sigma_q.
\]
It follows that when \( Q^* > Q_c \),
\[
\frac{d(Q^* - Q_c)}{ds_q} = \frac{dQ^*}{ds_q} - \frac{dQ_c}{ds_q} = (Q^* - Q_c)/\sigma_q > 0;
\]
and when \( Q^* < Q_c \),
\[
\frac{d(Q^* - Q_c)}{ds_q} = \frac{dQ^*}{ds_q} - \frac{dQ_c}{ds_q} = (Q_c - Q^*)/\sigma_q < 0.
\]
In both cases \( |Q^* - Q_c| \) increases in \( \sigma_q \).

(d) Since \( \partial \Pi / \partial f_c = -1 \), we obtain that
\[
\frac{dQ^*}{df_c} = \frac{\partial \Pi / \partial f_c}{\partial \Pi / \partial Q^*} = \frac{1}{\partial \Pi / \partial Q^*} > 0.
\]
and
\[
\frac{dQ^*}{df_c} = \frac{\sigma_q}{\phi(q^*_k)} > 0, \quad \text{where} \quad \sigma_q = (Q_c - \mu_q)/\sigma_q.
\]
Thus, increases in \( \sigma_q \) lead to increases in both \( Q^* \) and \( Q_c \).

\( \Box \)

**Proof of Proposition 1**

**Proof.** A. Preliminary results:

From Eq. (6), we obtain
\[
\frac{d}{dy} P(Y) = (1 + r_f)^{-1} \left\{ r - a(1 + r_f) - (r - s) \right\} \times \left[ (Y - Y) g(Y) + \int_0^Y g(Z) dZ \right]
\]
\[ - \frac{d}{dy} \left[ \left( Y - Y \right) g(Y) - \int_0^Y g(Z) dZ \right]
\]
\[ = (r - s + d) \delta_{MZ} \phi_Y(\frac{Y - \mu_Z}{\sigma_Z}) \cdot \frac{1}{\sigma_Z}
\]
\[ = (1 + r_f)^{-1} \left\{ r - a(1 + r_f) + d \right\} - (r - s + d) \int_0^Y g(Z) dZ
\]
\[ - (r - s + d) \delta_{MZ} \phi_Y(\frac{Y - \mu_Z}{\sigma_Z}) \cdot \frac{1}{\sigma_Z}
\]
\[ = (1 + r_f)^{-1} \left\{ r - a(1 + r_f) + d \right\} - (r - s + d) \int_0^Y g(Z) dZ
\]
\[ - (r - s + d) \delta_{MZ} \phi_Y(\frac{Y - \mu_Z}{\sigma_Z}) \cdot \frac{1}{\sigma_Z}
\]
\[ = (1 + r_f)^{-1} \left\{ r - a(1 + r_f) + d \right\} - (r - s + d)
\]
\[ \times \left[ \phi_Y(\frac{Y - \mu_Z}{\sigma_Z}) + \delta_{MZ} \phi_Y(\frac{Y - \mu_Z}{\sigma_Z}) \right]
\]
\[ = (1 + r_f)^{-1} \left\{ r - a(1 + r_f) + d \right\} - (r - s + d)
\]
\[ \times \left[ \phi_Y(\frac{Y - \mu_Z}{\sigma_Z}) + \delta_{MZ} \phi_Y(\frac{Y - \mu_Z}{\sigma_Z}) \right]
\]
\[ (A.5)
\]

From Lemmas 1 and 2, we know \( \frac{d}{dy} P(Y) \) is positive when \( Y \in [Y_{min}, Q^*] \). It follows that \( P(Y) \) is an increasing function when \( Y \in [Y_{min}, Q^*] \) until \( \frac{d}{dy} P(Y) = 0 \) at \( Y = Q^* \). We will explore conditions under which \( P(Y) \) is concave.

If \( \delta_{MZ} = 0 \), the firm aims to maximize its expected profit, and we have that \( \frac{d^2}{dy^2} P(Y) < 0 \) when \( Y \in [Y_{min}, Q^*] \), meaning that \( P(Y) \) is concave. If \( \delta_{MZ} \neq 0 \), we have that
\[
\frac{d^2}{dy^2} P(Y) = -(1 + r_f)^{-1} (r - s + d) \phi_Y(\frac{Y - \mu_Z}{\sigma_Z})
\]
\[ \times \left[ \phi_Y(\frac{Y - \mu_Z}{\sigma_Z}) \right]
\]
\[ = -(1 + r_f)^{-1} (r - s + d) \phi_Y(\frac{Y - \mu_Z}{\sigma_Z})
\]
\[ \times \left[ \frac{1}{\sigma_Z} \right]
\]
\[ = -(1 + r_f)^{-1} (r - s + d) \phi_Y(\frac{Y - \mu_Z}{\sigma_Z})
\]
\[ \cdot \left[ 1 - \delta_{MZ} \phi_Y(\frac{Y - \mu_Z}{\sigma_Z}) \right]
\]
\[ (A.4)
\]

We cannot prove that \( P(Y) \) is concave everywhere. However, note that Eq. (A.4) in the Proof of Lemma 2 shows that
\[
\frac{d^2}{dy^2} S(Q^*) = -(1 + r_f)^{-1} (r - s + d) \phi_Y(\frac{Q^*_k}{\sigma_q})
\]
\[ \cdot \left[ 1 - \delta_{MZ} ^2 \delta_{MZ} ^2 + \frac{Q^*_k}{\sigma_q} \right]< 0.
\]

When \( \delta_{MZ} > 0 \), we have from \( Y < Q^* \) that
\[
1 - \delta_{MZ} ^2 \delta_{MZ} ^2 + \frac{Q^*_k}{\sigma_q} > 1 - \delta_{MZ} ^2 \delta_{MZ} ^2 + \left( \frac{Q^*_k}{\sigma_q} \right) > 0.
\]
and hence \( \frac{d^2}{dy^2} P(Y) < 0 \) when \( Y \in [Y_{min}, Q^*] \).

When \( \delta_{MZ} < 0 \) and \( Y \in [Y_{min}, Q^*] \), if
\[
Y > \mu_Z + \frac{\sigma_Z}{\delta_{MZ} ^2}
\]
then it is guaranteed that
\[
1 - \delta_{MZ} ^2 \delta_{MZ} ^2 + \frac{Q^*_k}{\sigma_q} > 0.
\]
Hence, we arrive at a sufficient condition for \( \frac{d^2}{dy^2} P(Y) < 0 \), namely
\[
Y > \mu_Z + \frac{\sigma_Z}{\delta_{MZ} ^2}.
\]
Such condition leads to Proposition 1(c).

B. Proof of Proposition 1(a)

Since \( P(Y) \) is increasing when \( Y \in [Y_{min}, Q^*] \) and \( \delta_{MZ} > 0 \), we can proceed to show one property of \( S(Q^*) \). Let
\[
\bar{P}(Y) = \begin{cases} P(Y), & \text{if} \; Y < Q^*; \\ P(Q^*), & \text{if} \; Y \geq Q^*. \end{cases}
\]
\( \bar{P}(Y) \) is an increasing function when \( Y \in [Y_{min}, Q^*] \) and flat otherwise.

Based on Eq. (7), the maximum increase in firm value before mean-capacity improvement is
\[
S(Q^*)|_{Y=0} = \int_0^\infty \bar{P}(Y) f(Y) dY.
\]
Shifting the p.d.f. of the random capacity \( f(Y) \) by \( \Delta Y > 0 \), we have the firm value after mean-capacity improvement:
\[
S(Q^*, \Delta Y) = \int_0^\infty \bar{P}(Y + \Delta Y) f(Y) dY = \int_0^\infty \bar{P}(Y) f(Y) dY.
\]
It follows that
\[
\frac{dS(Q^*, \Delta Y)}{d\Delta Y} = \int_0^{+\infty} \frac{d}{d\Delta Y} \mathcal{P}(Y + \Delta Y) f(Y) dY \\
= \int_0^{+\infty} \mathcal{P}'(Y + \Delta Y) f(Y) dY > 0.
\]
In other words, shifting \( f(Y) \) to the right improves firm value \( S(Q^*) \).

The proof of Proposition 1(a) is based on the fact that \( \mathcal{P}(Y) \) is a non-decreasing function and an increasing function at certain intervals.

When \( \mathcal{P}(Y) \) is concave, we proceed to show additional properties of \( S(Q^*) \) in Proposition 1(b). The proof of Proposition 1(b) comes from the fact that \( \mathcal{P}(Y) \) is a concave, non-decreasing function and an increasing function at certain intervals.

Proof. From the lines above, we obtain
\[
\frac{d^2\bar{S}(Q^*, b)}{db^2} = \int_0^{+\infty} \frac{d}{db} \left[ \frac{\mathcal{P}(u - \mu_Y)}{b} + \mu_Y \right] \cdot \left[ -\left( u - \mu_Y \right) / b^2 \right] f(u) du \\
= \frac{1}{b^2} \int_0^{+\infty} \left( u - \mu_Y \right) f(u) du = 0
\]
for some \( b > 0 \). We conclude that \( \frac{d^2\bar{S}(Q^*, b)}{db^2} \leq 0 \), which completes the proof.

E. Proof of Proposition 1(b)(iii)
From the lines above, we obtain
\[
\frac{d^2\bar{S}(Q^*, b)}{db^2} = \int_0^{+\infty} \frac{d^2}{db^2} \left[ \frac{\mathcal{P}(u - \mu_Y)}{b} + \mu_Y \right] f(u) du \\
= \int_0^{+\infty} \left[ \mathcal{P}'(u - \mu_Y) / b + \mu_Y \right] \cdot \left[ -\left( u - \mu_Y \right) / b^2 \right] f(u) du \\
+ \left[ \mathcal{P}(u - \mu_Y) / b + \mu_Y \right] \cdot \left[ -2(u - \mu_Y)^2 / b^4 \right] f(u) du \\
\leq 2 \int_0^{+\infty} \left[ \mathcal{P}(u - \mu_Y) / b + \mu_Y \right] \cdot \left[ -\left( u - \mu_Y \right) / b^2 \right] f(u) du \\
< 0.
\]

Compared to the original distribution \( f(Y) \), \( \tilde{f}(Y) \) is unchanged when \( b = 1 \) and shrinks towards the mean when \( b > 1 \).

Denoting \( \sigma^2 = \int_0^{+\infty} (Y - \mu_Y)^2 f(Y) dY \), we arrive at the variance of random capacity after capacity-variance reduction:
\[
\tilde{\sigma}^2 = \int_0^{+\infty} (Y - \mu_Y)^2 b f(Y) dY = b \int_0^{+\infty} (Y - \mu_Y)^2 f(u) du = \frac{\sigma^2}{b^2}.
\]
Thus, the transformation \( \tilde{f}(Y) \) changes the variance of the random capacity in proportion to \( \frac{1}{b^2} \).

D. From the lines above, it follows that \( \tilde{S}(Q^*, b) \) has an upper-bound \( \mathcal{P}(\mu_Y) \) that follows from
\[
\lim_{b \to +\infty} \int_0^{+\infty} \mathcal{P}(u - \mu_Y) / b + \mu_Y \right] f(u) du = \int_0^{+\infty} \mathcal{P}(\mu_Y) f(u) du = \mathcal{P}(\mu_Y).
\]

Proof of Corollaries 2, 3 and 4

Proof. To begin with, we analyze how \( S(Q^*) \) changes as the mean-capacity improvement \( \Delta Y \) increases using Eq. (A.5). With mean-capacity improvement \( \Delta Y \), we have
\[
S(Q^*, \Delta Y) = J_0^{+\infty} \mathcal{P}(Y) f(Y - \Delta Y) dY = J_0^{+\infty} \mathcal{P}(Y + \Delta Y) f(Y) dY
\]
and
\[
\frac{dS(Q^*, \Delta Y)}{d\Delta Y} = \int_0^{+\infty} \mathcal{P}'(Y + \Delta Y) f(Y) dY \\
= \int_0^{+\infty} \mathcal{P}'(Y) f(Y - \Delta Y) dY \\
= \int_0^{Q^*} \mathcal{P}'(Y) f(Y - \Delta Y) dY \\
= (1 + r)^{-1} \int_0^{Q^*} \left[ \mathcal{P}' \left( \frac{Y - \mu_Y}{\sigma^2} \right) - \delta \mathcal{P}' \left( \frac{Y - \mu_Y}{\sigma^2} \right) \right] f(Y - \Delta Y) dY.
\]
Note that increasing $\frac{\partial S(Q^*, \Delta Y)}{\partial \Delta Y}$ is a sufficient condition for increasing the benefit of mean-capacity improvement, since

$$S(Q^*, \Delta Y) - S(Q^*) = \int_0^{\Delta Y} \frac{\partial S(Q^*, \Delta Y)}{\partial \Delta Y} \, d\Delta Y.$$ 

In the remainder of this proof, we discuss how various parameter changes impact $\frac{\partial S(Q^*, \Delta Y)}{\partial \Delta Y}$ to analyze the benefit of mean-capacity improvements. Similarly, we also discuss how various parameters impact $\frac{\partial S(Q^*, b)}{\partial b}$ to analyze the benefit of capacity-variance reductions.

1. Proof of Corollary 2
   (a) (i) Increasing $\delta_{MZ}$ reduces both $[\varepsilon - \Phi(Y_{YZ}^{1/2}) - \delta_{MZ} \Phi(Y_{YZ}^{1/2})]$ and $Q^*$ (Corollary 1), and hence reduces $\frac{\partial S(Q^*, \Delta Y)}{\partial \Delta Y}$.
   (a) (ii) When $\delta_{MZ} > 0$, increasing $\varepsilon_R$ reduces both $[\varepsilon - \Phi(Y_{YZ}^{1/2}) - \delta_{MZ} \Phi(Y_{YZ}^{1/2})]$ and $Q^*$ (Corollary 1), and hence reduces $\frac{\partial S(Q^*, \Delta Y)}{\partial \Delta Y}$.
   When $\delta_{MZ} < 0$, decreasing $\varepsilon_R$ increases both $[\varepsilon - \Phi(Y_{YZ}^{1/2}) - \delta_{MZ} \Phi(Y_{YZ}^{1/2})]$ and $Q^*$ (Corollary 1), and hence increases $\frac{\partial S(Q^*, \Delta Y)}{\partial \Delta Y}$.

(b) (i) Based on Eq. (A.6), we have that

$$\frac{d}{d\delta_{MZ}} \left( \frac{dS(Q^*, b)}{dB} \right) = \frac{d}{d\delta_{MZ}} \left( \int_{0}^{\infty} \frac{p\left( u - \mu_Y \right)}{b + \mu_Y} \right) \times \left( -\frac{u - \mu_Y}{b^2} \right) \, f(u) \, du$$

$$= \left( 1 + r_f \right)^{-1} \frac{b_{Q^*} - (b - 1) \mu_Y}{b_{Q^*}} \frac{d}{d\delta_{MZ}} \int_{0}^{\infty} \frac{p\left( u - \mu_Y \right)}{b + \mu_Y} \, du$$

$$= \left( 1 + r_f \right)^{-1} \frac{b_{Q^*} - (b - 1) \mu_Y}{b_{Q^*}} \times \left( -\frac{u - \mu_Y}{b^2} \right) \, f(u) \, du$$

$$= \left( 1 + r_f \right)^{-1} \frac{b_{Q^*} - (b - 1) \mu_Y}{b_{Q^*}} \times \left( -\frac{u - \mu_Y}{b^2} \right) \, f(u) \, du.$$

When $Q^* \leq \mu_Y$, it follows that $b_{Q^*} - (b - 1) \mu_Y \leq \mu_Y$ and we arrive at $\frac{d}{d\delta_{MZ}} \left( \frac{dS(Q^*, \Delta Y)}{\partial \Delta Y} \right) < 0$. (b) (ii) Using the transformation $Y = (u - \mu_Y)/b + \mu_Y$, we have that

$$\frac{d}{d\varepsilon_R} \left( \frac{dS(Q^*, b)}{dB} \right) = \frac{d}{d\varepsilon_R} \int_{0}^{b_{Q^*} - (b - 1) \mu_Y} p\left( \frac{u - \mu_Y}{b} + \mu_Y \right) \times \left( -\frac{u - \mu_Y}{b^2} \right) \, f(u) \, du$$

$$= \left( 1 + r_f \right)^{-1} \frac{b_{Q^*} - (b - 1) \mu_Y}{b_{Q^*}} \times \left( -\frac{u - \mu_Y}{b^2} \right) \, f(u) \, du$$

$$= \left( 1 + r_f \right)^{-1} \frac{b_{Q^*} - (b - 1) \mu_Y}{b_{Q^*}} \times \left( -\frac{u - \mu_Y}{b^2} \right) \, f(u) \, du.$$

When $Q^* \leq \mu_Y \Rightarrow b_{Q^*} - (b - 1) \mu_Y \leq \mu_Y$, we arrive at that $\frac{d}{d\varepsilon_R} \left( \frac{dS(Q^*, \Delta Y)}{\partial \Delta Y} \right) < 0$.

2. Proof of Corollary 3: It follows directly from Proposition 1. Since mean-capacity improvements exhibit diminishing returns when (1) $\delta_{MZ} \in [0, 1]$ or (2) $\delta_{MZ} \in [1, -1]$ and $\gamma_{min} > \mu_Z + \frac{\sigma_{MZ}}{b_{Q^*}}$, lower $\mu_Y$ (i.e., higher capacity scariness $\rho = \mu_Z/\mu_Y$) means higher benefits from mean-capacity improvement. Since capacity-variance reductions exhibit diminishing returns when (1) $\delta_{MZ} \in [0, 1]$ or (2) $\delta_{MZ} \in [1, -1]$ and $\gamma_{min} > \mu_Z + \frac{\sigma_{MZ}}{b_{Q^*}}$, lower $\mu_Y$ (i.e., higher capacity scariness $\rho = \mu_Z/\mu_Y$) means higher benefits from capacity-variance reductions.

3. Proof of Corollary 4
   First, increasing $\varepsilon_R$ increases both $[\varepsilon - \Phi(Y_{YZ}^{1/2}) - \delta_{MZ} \Phi(Y_{YZ}^{1/2})]$ (which is positive when $Y \in (0, Q^*)$) and $Q^*$ (Corollary 1), and hence increases $\frac{\partial S(Q^*, \Delta Y)}{\partial \Delta Y}$.

Second, using the transformation $Y = (u - \mu_Y)/b + \mu_Y$, we have that

$$\frac{d}{d\varepsilon_R} \left( \frac{dS(Q^*, b)}{dB} \right) = \frac{d}{d\varepsilon_R} \int_{0}^{b_{Q^*} - (b - 1) \mu_Y} p\left( \frac{u - \mu_Y}{b} + \mu_Y \right) \times \left( -\frac{u - \mu_Y}{b^2} \right) \, f(u) \, du$$

$$= \left( 1 + r_f \right)^{-1} \frac{b_{Q^*} - (b - 1) \mu_Y}{b_{Q^*}} \times \left( -\frac{u - \mu_Y}{b^2} \right) \, f(u) \, du$$

$$= \left( 1 + r_f \right)^{-1} \frac{b_{Q^*} - (b - 1) \mu_Y}{b_{Q^*}} \times \left( -\frac{u - \mu_Y}{b^2} \right) \, f(u) \, du.$$

If $Q^* \leq \mu_Y \Rightarrow b_{Q^*} - (b - 1) \mu_Y \leq \mu_Y$, we have that $\frac{d}{d\varepsilon_R} \left( \frac{dS(Q^*, \Delta Y)}{\partial \Delta Y} \right) > 0$. Otherwise, if $Q^* > \mu_Y \Rightarrow b_{Q^*} - (b - 1) \mu_Y > \mu_Y$, we have

$$\frac{d}{d\varepsilon_R} \left( \frac{dS(Q^*, b)}{dB} \right) = \left( 1 + r_f \right)^{-1} \frac{b_{Q^*} - (b - 1) \mu_Y}{b_{Q^*}} \times \left( -\frac{u - \mu_Y}{b^2} \right) \, f(u) \, du > 0.$$

In both cases, we have that $\frac{d}{d\varepsilon_R} \left( \frac{dS(Q^*, \Delta Y)}{\partial \Delta Y} \right) > 0$. □
References


